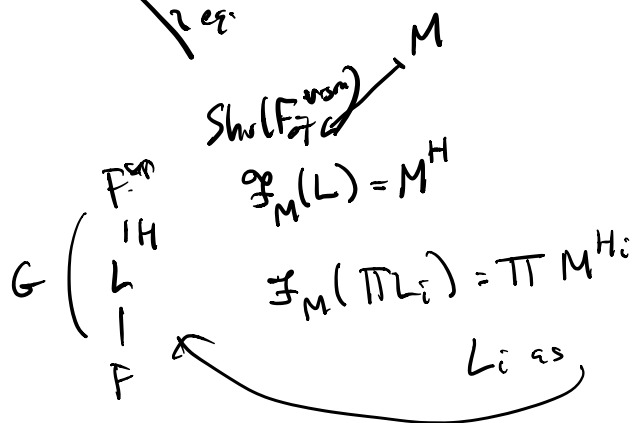


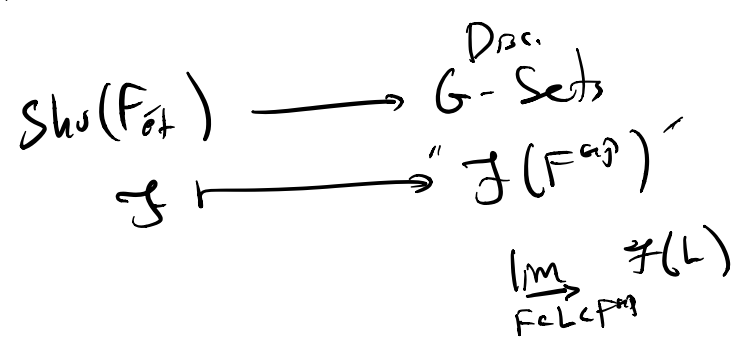
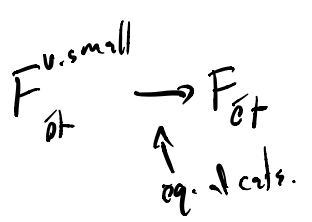
Facts (Etale cohomology of a field)

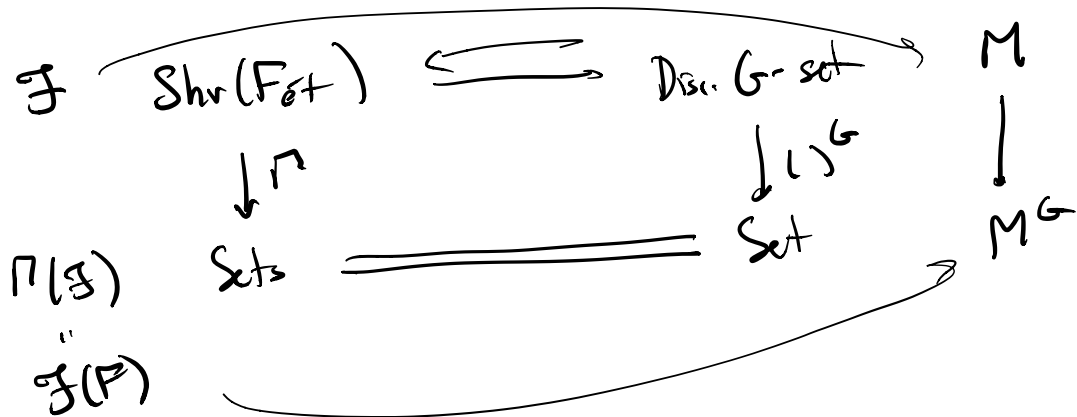
Eq. of cats F field $F_{\text{ét}} = \varprojlim_{\text{small}} k$ site of F
 $G = \text{Gal}(F^{\text{sep}}/F)$
 $\mathfrak{S} \longrightarrow \mathfrak{S}(L) \xrightarrow{\text{Gal } L/F} \text{ob}(F_{\text{ét}}) = (\text{Spec } L \rightarrow \text{Spec } F)$
 $\text{Shv}(F_{\text{ét}}) \longleftrightarrow \text{Discrete } G\text{-Sets}$
 $L = \prod_{i=1}^n L_i$
 L_i/F sep. ext.



cons: not triv. L

def $F_{\text{ét}}^{\text{v.small}}$
 " as in $F_{\text{ét}}$ but
 w/ only L/F as here





$$\text{Ab Shv}(F_{\text{ét}}) \rightleftarrows \text{Disc } G\text{-Mod}$$

$$R\Gamma = R(\cdot)^G$$

$$H^i(F_{\text{ét}}, \mathcal{F}) \xrightarrow{\sim} H^i(\text{Gal}(F^{\text{sep}}/F), \mathcal{F}(F^{\text{sep}}))$$

\uparrow choosing sep closure gives a canonical ism

$$\mathcal{F} = \mu_n \quad (n, \text{char } X) \quad X \text{ scheme.}$$

$$1 \rightarrow \mu_n \rightarrow G_m \xrightarrow{\cdot^n} G_m \rightarrow 1$$

sheaves on big or small étale sites on X

$$\mu_n(y) = \{ t \in \mathcal{O}_y^* \mid t^n = 1 \}$$

$$G_m(y) = \mathcal{O}_y^*$$

$$G_m = G_{m,x} = \text{Spec } A'_x \rightarrow X$$

locally $X = \text{Spec } R$

$$\text{Spec } R[t, t^{-1}]$$

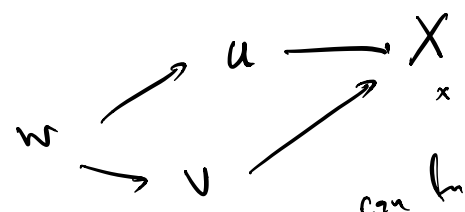
$$\mu_n \hookrightarrow G_m$$

$$\mu_{n,R} = \text{Spec } R[t] / (t^n - 1)$$

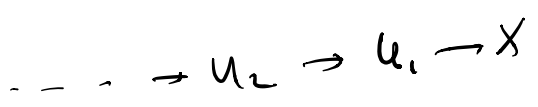
Points in étale site:

if X is a scheme, $x \in X$ scheme theoretic pt
 consider "stalks at x "
 limits of "covers at x "

limits of étale morphisms w/ x in image.
 exactness for sheaves on X only needs to be checked
 for sections in each of these limits for each x .



can find a "limiting"
 cofinal \nearrow
 family of covers



with all
 U_i 's connected
 ind.

in limit, get a locally.

Def. A strictly local \mathcal{O}_y is a Henselian local \mathcal{O}_y w/ separably closed residue field.

Hensel's lemma holds

How to think of this:

take Zariski local \mathcal{O}_y $R = \mathcal{O}_{X,x}$

Force Hensel to hold:

for each poly $f \in R[x]$ w/ $\bar{f} \in \bar{R}/\mathfrak{m}[x]$ sep., α a root of \bar{f} adjoin a root a of f restricting to α .

get a Henselian $\mathcal{O} \subset R^h$

res ext $k^{\text{sep}}/k \rightarrow \text{res ext } R^h/R^h$
unram.

$R^h = \text{strictly local } \mathcal{O}$ of $\text{st } x$.

way back

Kummer seq

$$0 \rightarrow H^0(X, \mu_n) \rightarrow H^0(X, \mathcal{O}_m) \rightarrow H^0(X, \mathcal{O}_m)$$

$$H^1(X, \mu_n) \rightarrow H^1(X, \mathcal{O}_m) \rightarrow H^1(X, \mathcal{O}_m)$$

" Pic X " Pic X

$$0 \rightarrow \mathcal{O}_X(X)/n \rightarrow H^1(X, \mu_n) \rightarrow \text{Pic } X[n] \rightarrow 0$$

$$X = \text{Spec } F$$

$$F^*/(F^*)^n \cong H^1(F, \mu_n)$$