

## Graph Theory, Spring 2016, Homework 4

1. Let  $G$  be a graph and  $e \in E_G$ . If  $e$  is incident to vertices  $u, v$  (not necessarily distinct), let  $G/e$  denote the graph  $G/\{u, v\}$ .  
Show that if  $G$  is a graph is  $k$ -connected, then  $G/e$  is  $(k - 1)$ -connected.
2. Show that if a simple connected graph contains a cycle, then it must contain at least 3 distinct spanning subtrees.
3. Suppose that a simple **connected** graph  $G$  contains 10 vertices and 11 edges. Show it is possible for number of cycles in  $G$  to be 2 or 3 but that no other value is possible.
4. A graph  $G$  is called bipartite if we may partition the set of vertices into two nonempty disjoint parts  $V(G) = V_1 \cup V_2$ , such that whenever two vertices  $v, w$  are in the same part  $V_i$ , we have that  $v$  and  $w$  are nonadjacent. In other words, the only edges in the graph are those which connect vertices in different parts. Show that if  $G$  is a tree with at least 2 vertices then  $G$  is bipartite.