Graph Theory, Spring 2016, Homework 8

1. Let D be a digraph and $x, y \in V_D$. We say that $S \subset A_D$ is an (x, y)-arrow cut if $D - S$ contains no directed (x, y)-paths. We say that is minimal if there is no (x, y)-arrow cut S' with S' a proper subset of S.

   If X, Y are subsets of V_D, we write $[X, Y]$ to denote the set of all arrows of A_D whose source is a vertex in X and whose target is a vertex in Y.

   Show that if K is a minimal (x, y)-arrow cut, then we can partition the vertices of V_D as $V_D = X \cup Y$ with $x \in X, y \in Y$ and $X \cap Y = \emptyset$, and with $K = [X, Y]$.

2. Suppose that D is as in the previous problem, and that K is a minimal (x, y) arrow cut. Give an algorithm for producing the sets X and Y as above.

3. (bonus points) Suppose that D is as above. Characterize which partitions $X \cup Y = V_D$ correspond to minimal (x, y) cuts.