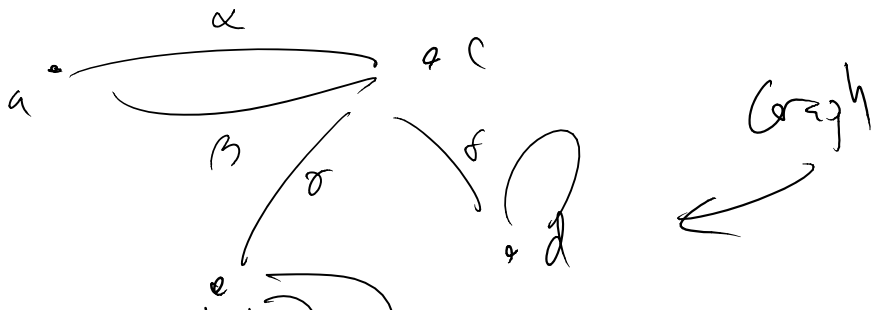
Lecture 1: What's a graph?

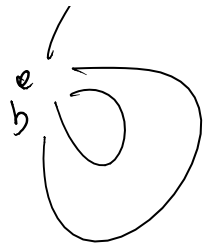
Tuesday, January 12, 2016 12:28 PM

Text?
Bondy & Murty

Grady scale:
A: really understand everything
B: shown good competence
C: good evidence that you are learning something.
D: weaker evidence
F: no evidence.

Highly recommended reading: Pearls in Graph Theory (Dover)





Formally

Def A graph G is an ordered triple (V, E, ψ) consisting of

- a set V of "vertices"
- a set E of "edges"
- a function $\psi : E \longrightarrow$ Unordered pairs of vertices

("incidence")

Set theory Notation Aside

S set, $S \times S = \{(s_1, s_2) \mid s_i \in S\}$ "ordered pairs"

A multiset S is a pair (S, m)

(S: set; m: function $m : S \longrightarrow \mathbb{Z}_{\geq 0}$)

$\{a, b, c\}$

cbaacb ... abbacc ↪ multiset containing a twice, b twice, c twice

cbaacb

b twice
c twice

$(\{a,b,c\}, m)$

$m(a) = 2 = m(b) = m(c)$

if T a set, $S = (S, m)$ is a multiset, we say

$S \subset T$ if $S \subset T$.

$\#S = \sum_{s \in S} m(s)$ "cardinality of S"

Def an unordered pair of element of a set S is a multisubset of S of cardinality 2.

Notation: if $G = (E, V, \psi)$

write $E_G = E$ $V_G = V$ $\psi_G = \psi$

ex: given graphs G, H, K

V_H = vertices of H ψ_K = incidence fcn for K.

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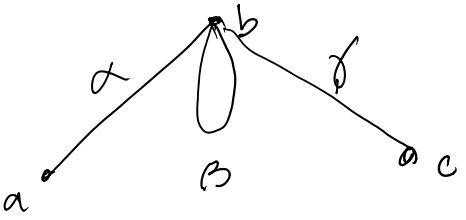
ex: G, $V_G = \{a, b, c\}$, $E_G = \{\alpha, \beta, \gamma\}$

$\psi_G(\alpha) = ab$ $\psi_G(\beta) = bb$ $\psi_G(\gamma) = bc$

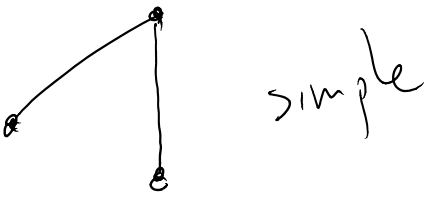


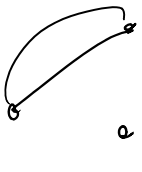
Def: A loop is an edge incident to only one vertex

Def we say that an edge $e \in E_G$ & vertex $v \in V_G$ are incident if $v \in \psi_G(e)$

Def A graph G is called simple if

- it has no loops
- every pair of vertices are incident to at most one edge.





Examples

Scheduling Problems:

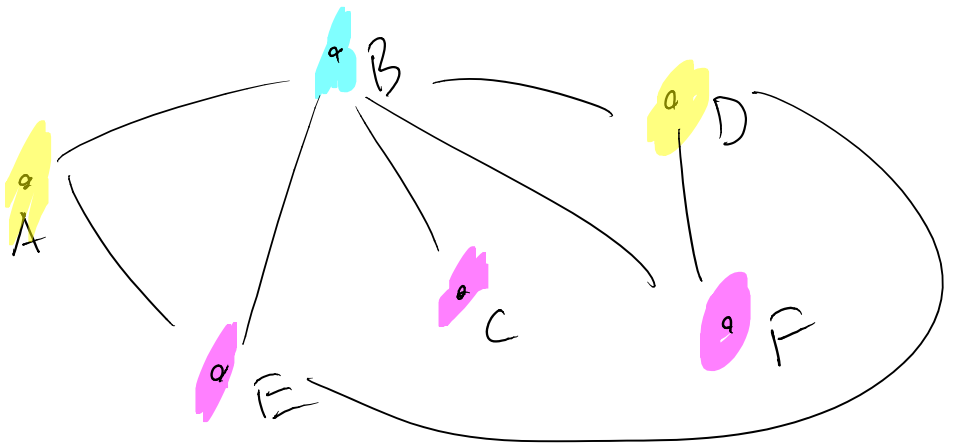
If 1 is the prerequisite

Scheduling Problems

jobs, resources that jobs require

vertices = jobs

edges = conflicts (simple graph)



"proper vertex coloring"

Wedding table planning

some guests, some people don't get along.

want to assign people to tables w/ people they get along with

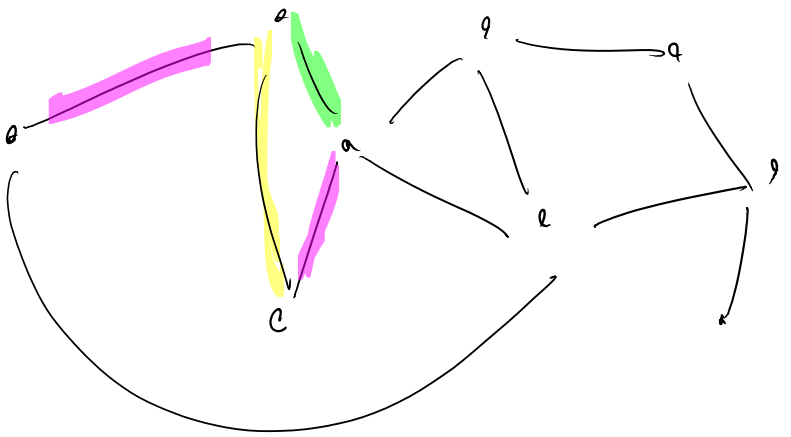
- vertices = people
- edges = dislikes

Tournaments

teams in competition, disjoint pairings can play simultaneously

how many rounds to make everyone play w/ eachother?

- vertices: teams
- edges: games which need to be played



"proper edge coloring"