Lecture 10: connectivity, part 3

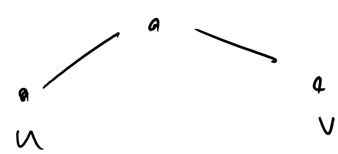
Thursday, February 11, 2016 12:35 PM

<u>Thm</u> (Menger) if G connected, u,v nonadjacent then $p(u,v) = \kappa(u,v)$.

<u>Pf</u>: First, $p(u,v) \le \kappa(u,v)$ since if there are p int. disjoint paths $P_1, P_2, \ldots, P_p$ from u to v. and we remove less than p vertices $v_1, \ldots, v_\ell$, at least one path P_i will still be in $G - \{v_1, \ldots, v_\ell\}$

Interesting direction: to show $\kappa(u,v) \le p(u,v)$

Induction on $e(G)$.

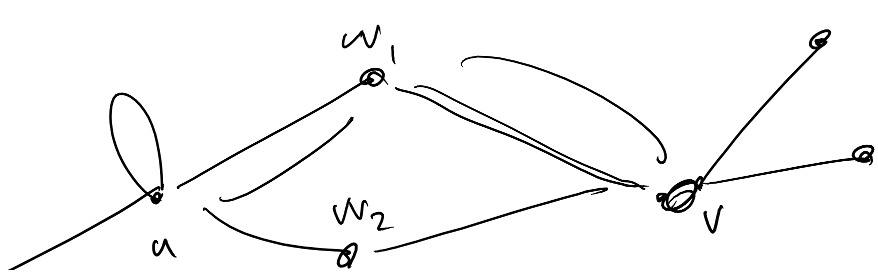


$e(G) = 0$ ×

$= 1$ ×

$= 2$ ✓

<u>Case</u>: suppose every edge in graph is incident either to u or v.



in this case, have a list of vertices $w_1, w_2, \dots, w_p$ s.t. every path from u to v is of the form

$u e w_i f v$ same e's f's.

see that in this case, have collection of p int. disj paths, and $p = K(u,v)$ ✓ case done.

<u>Other Case</u>: $\exists$ edge e incident to neither u or v

suppose e a loop. remove it, done by induction.

other other: e incident to a, b distinct vertices.

Consider $G' = G - e$

$p_{G'}(u,v) \le p_G(u,v)$ $\quad K_G(u,v) \le K_{G'}(u,v) + 1$

if $x_1, \dots, x_k$ is a min'l u-v vertex cut in G'

then $x_1, \dots, x_k, a$ is a vertex cut in G

$$G - \{x_1, \dots, x_k, a\} \subseteq (G - e) - \{x_1, \dots, x_k\}$$

also, by induction, have $p_{G'}(u,v) = K_{G'}(u,v)$

$$p_G(u,v) \geq p_{G'}(u,v) = K_{G'}(u,v) \geq K_G(u,v) - 1$$

if $p_G(u,v) > p_{G'}(u,v)$ then $p_G(u,v) \geq K_G(u,v)$ and we're done.

so WLOG, can assume $p_G(u,v) = \underbrace{p_{G'}(u,v)}_{= K_{G'}(u,v)} = \underbrace{K_G(u,v)}_{= K}-1$