Lecture 3: subgraphs

Tuesday, January 19, 2016 12:02 PM

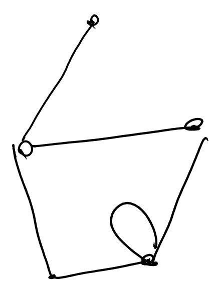
2016 Number theory REU "WADE Into Research"

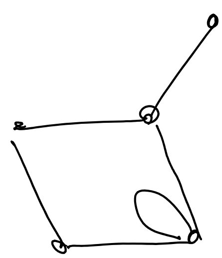
Feb 14 deadline

http://college.wfu.edu/mathreu/details/2015-summary.

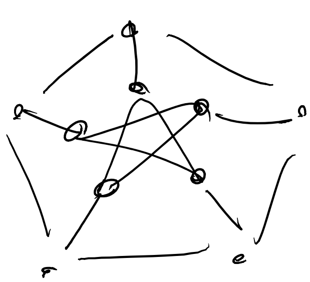
Practice w/ Isomorphism

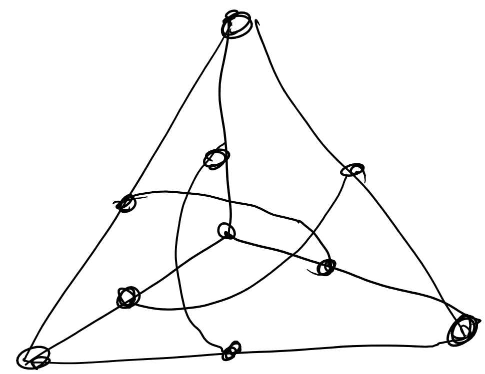
Prove that these graphs are not isomorphic





show these are isomorphic





Warm up

\# edges not same

if they are isomorphic, the isomorphism

$f: G \longrightarrow H$ would have

$f_E: E_G \longrightarrow E_H$ give a bijection between edges.

not isomorphic since some vertex goes to v via

$f_V: V_G \longrightarrow V_H$, and each vertex in G is incident to at least 1 edge. this edge via

$f_E: E_G \longrightarrow E_H$ would have have to map an edge incident to v, but there aren't any!

- Prove that degree is preserved by isomorphisms

- lemma: isomorphisms preserve adjacency.

Subgraphs

Def If G is a graph, a subgraph H of G (denoted $H < G$) is a graph such that $V_H \subseteq V_G$, $E_H \subseteq E_G$,

$$\psi_G|_{E_H} = \psi_H$$

Typical Question

Given G, and H graphs, can we find a subgraph of G which is isomorphic to H?

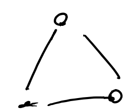
Def If G is a graph, $W \subseteq V_G$ any subset of vertices of G, we define $G[W]$ "the subgraph induced by W" to be the graph w/ vertices W.

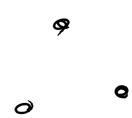
i.e. $V_{G[W]} = W$ & edges

$$E_{G[W]} = \{ e \in E_G \mid \psi_G(e) \subseteq W \}$$

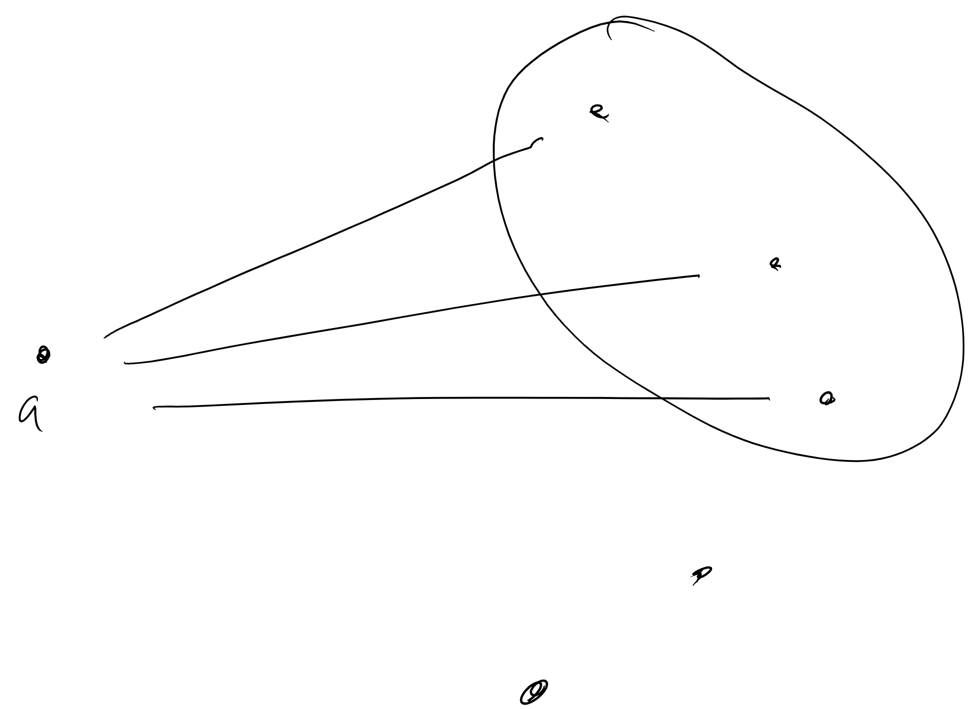
$$\psi_{G[W]} = \psi_G |_{G[W]} .$$

Example:

Suppose G is a simple graph with 6 vertices then either we can find 3 vertices $S = \{v_1, v_2, v_3\} \subset V_G$ s.t. $G[S] \cong$ 

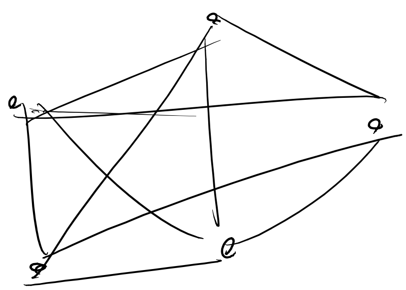
or can find S s.t. $G[S] \cong$ 

Pf:

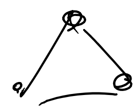


Def K_n "complete graph on n vertices"

$V_{K_n} = \{1, \ldots, n\}$ all edges (simple graph)



K_5

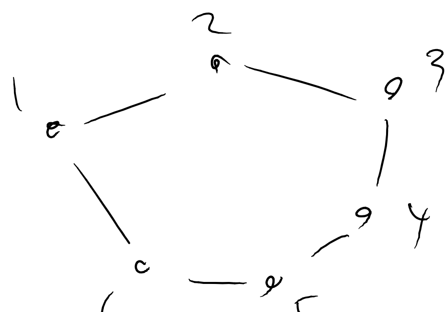


Def C_n "cycle graph on n vertices"

is the graph w/ $V_{C_n} = \{1, \ldots, n\}$ $n \geq 3$

and w/ i adj. to j if either

- they differ by 1
- they differ by $n-1$.



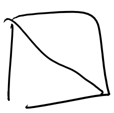
C_6 .

Def If G is a graph, an n-cycle in G is a subgraph of G which is isomorphic to C_n.

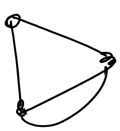
Def If G is a graph, an n-clique in G is a subset $S \subseteq V_G$ w/ $\# S = n$ s.t.

$G[S] \simeq K_n$.

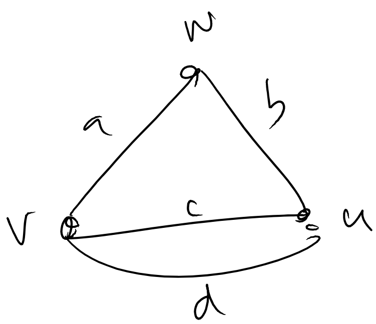
Practice: give an example of graph w/



- 4 vertices & exactly 3 cycles (simple)



- 3 vertices & exactly 2 cycles.



H_1: $V_{H_1} = \{u, v, w\}$
$E_{H_1} = \{a, b, c\}$

H_2: $V_{H_2} = V_{H_1}$
$E_{H_2} = \{a, b, d\}$
