Worksheet/Homework for Lecture 21

Suppose that G is a connected graph which has no odd cycles.

1. Show that if v and w are vertices and P_1, P_2 are internally disjoint $v - w$ paths, then either both P_1 and P_2 have even length, or they both have odd length.
2. Show that if v and w are vertices, then either every $v - w$ path has even length, or every $v - w$ path has odd length.

Hint: for 2, consider two paths, and consider their first point of intersection. Use induction on the length of the path, and part 1.