

## Meeting schedule

Meeting will open ~ 8:40 begin at 9  
"Formal" lecture portion end at or before 10:00

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Check questionnaire in email

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Syllabus has been revised (website)

exams: take home, open book, turned in on Sakai

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### Format:

will give ~10 minute videos Tues/Fridays  
(most of the time)

& worksheets

→ guide discussions by group  
meetings

Some weekly HW's.

via Sakai

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Graph theory & epidemiology

geographic/spatial distribution

discrete (cities) (houses)

etc

continuous  
 graph theory  
 vertices — locations  
 edges (lengths) — neighboring areas

graph theory used to give a language to  
 describe priority of areas & socio-economic  
 status  
 3 models  
 distribution of medical care.

Recent epidemiological paper

Q: when do we expect actual exponential growth in  
a disease outbreak?

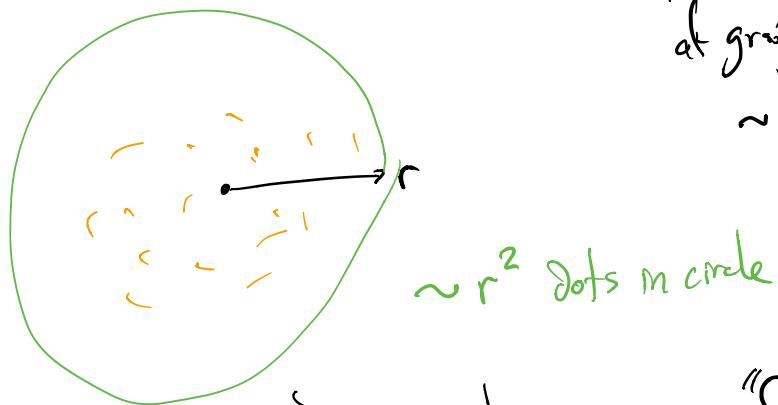
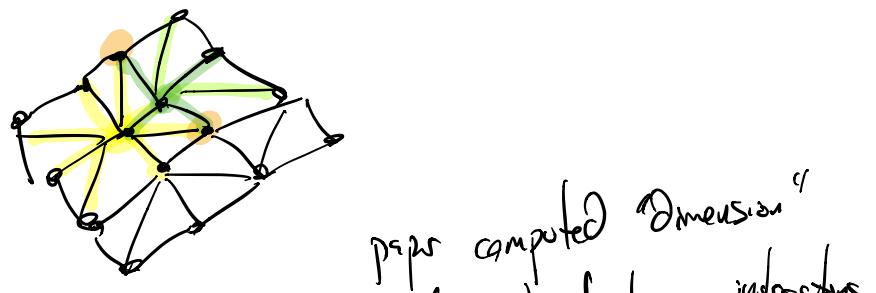
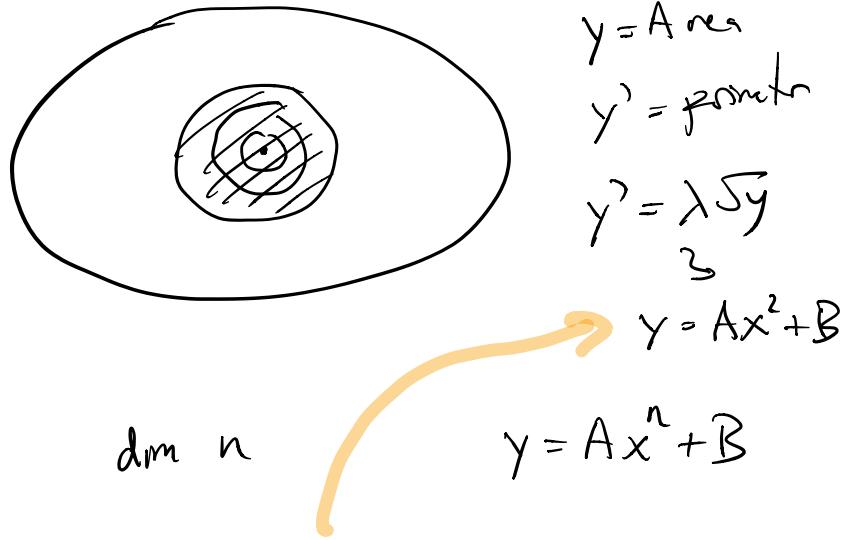
$$y' = \lambda y \rightarrow y = C e^{\lambda x}$$

assumption — "homogeneity"  
 at every pt in time,  
 infected people are in contact  
 w/ same # of uninfected.



$$y' = \lambda y$$

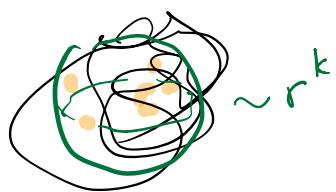
$$y = A x + B$$



crumple

"fractal dimension"

"Hausdorff dimension"



$\sim r^k$      $2 \leq k \leq 3$

Recall: if  $u, v \in V(G)$   $\delta(u, v) = \min$  length of a  $u-v$  path in  $G$

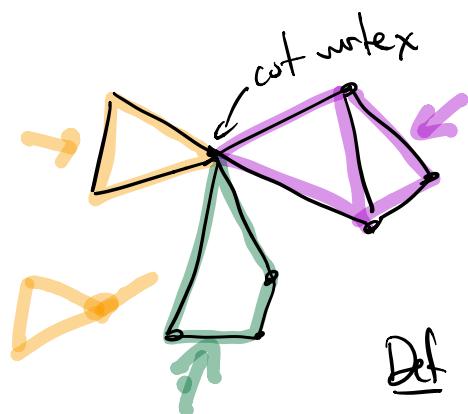
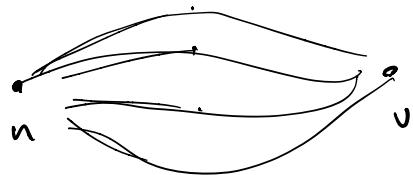
given  $v$ , can define  $B_r(v) = \{u \in V(G) \mid \delta(v, u) \leq r\}$

$\# B_r(v) \sim r^d$  some  $d = \text{"dimension"}$

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Goal: Menger's theorem

relates ideas of connectedness/separability  
with throughput/flow capacity

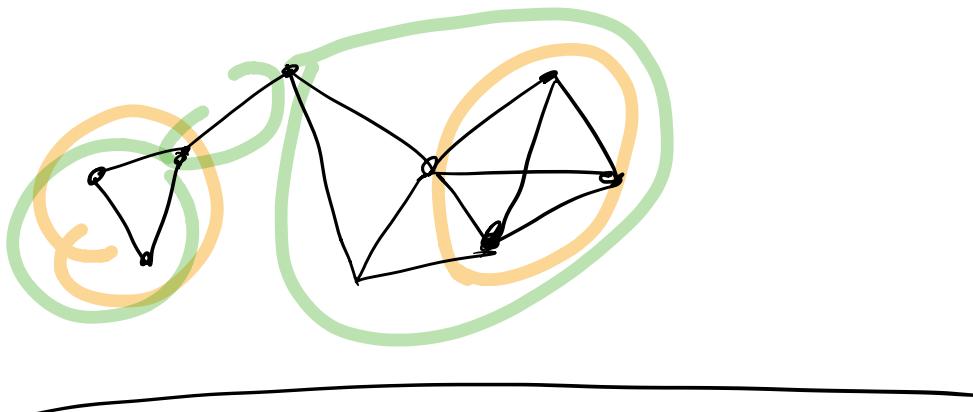
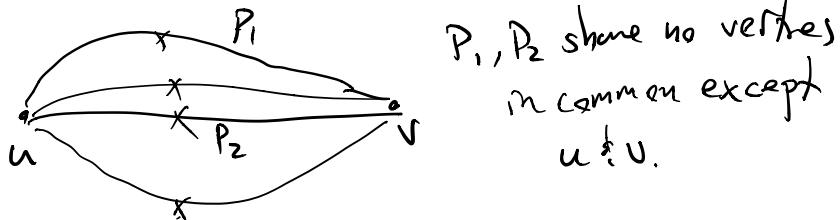


Vertex cut: (lecture 10)  
a subset  $S \subset V(G)$   
s.t.  $G - S$  is disconnected

Def a  $u-v$  separating set is a vertex cut  $S$  s.t.  $u, v$  on different components of  $G-S$  ( $u, v \notin S$ )

let  $u \neq v$  be nonadjacent

Theorem (Menger) The minimum size of a  $u-v$  separator set = the maximum # of mutually internally disjoint  $u-v$  paths.




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Previously :

Def: A block = maximal nonseparable subgraph

Thm: A graph is nonseparable if and only if every pair of vertices lies on a common cycle.

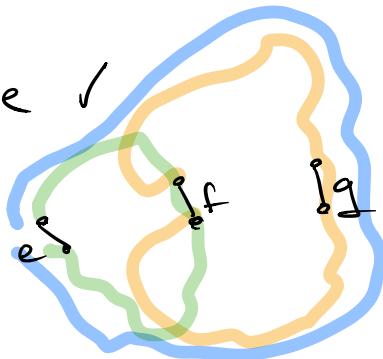
Defn an equivalence relation on edges of a graph:

we say  $e \sim f$  if  $e \stackrel{\text{def}}{\sim} f$  lie on a common cycle.  
 $e = f$  or

- reflexive?  $e \sim e$  ✓

- symmetric?  $e \sim f \Rightarrow f \sim e$  ✓

- transitivity? nontransit.



Magical Result:

equivalence classes  $\longleftrightarrow$  blocks

give maximal nonseparable subgraphs.

$$[e] = \{f \in E(G) \mid f \sim e\}$$

$e$  in  $H$ , nonseparable,  $e \sim f$  can add stuff to  $H$ , include  
 $f$ , to get a larger  
possibly

nonsep. sub-graph.

Conversely, can show any  $e, f$  in same block  
must be equivalent ( $e \sim f$ )