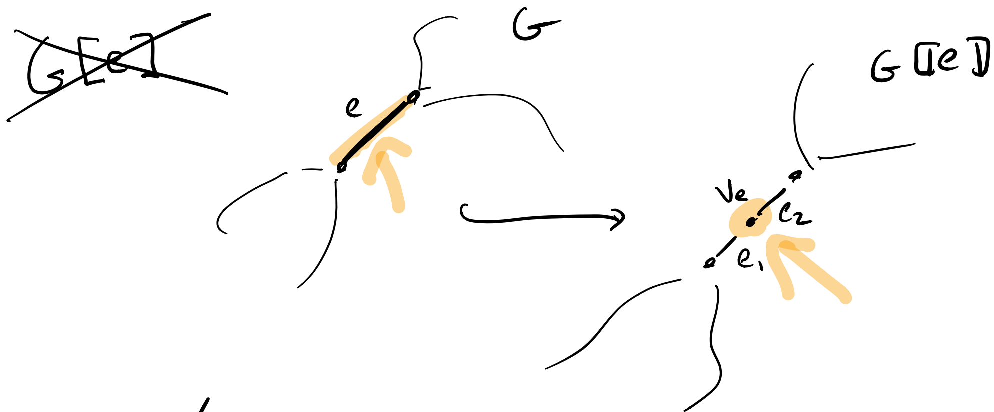
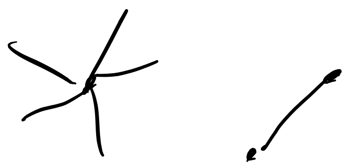
Problem: Suppose G a graph $e \in E(G)$.
Show that G is nonseparable if and only if $G[\![e]\!]$ is nonseparable.





Strategies:

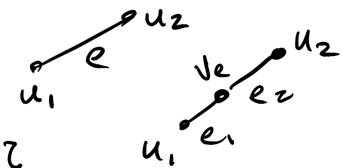
nonseparable $\leftrightarrow$ no cut vertices

nonseparable $\leftrightarrow$ every two vertices lie on a common cycle (if G has at least 3 vertices)

Ex: Suppose that every two vertices in $G[\![e]\!]$ lie on a common cycle. Will show every two vertices in G lie on a common cycle.

$$\left(G[\![e]\!] \text{ noncy} \overset{?}{\Leftarrow}\!\!\Rightarrow G \text{ ncy} \right)$$

So choose $v, w \in V(G)$



$$V(G[\![e]\!]) = \underset{v,w}{V(G)} \cup \{v_e\}$$

by assumption, $\exists$ a cycle $C \subset G[\![e]\!]$ in $G[\![e]\!]$ containing v & w.

$\Rightarrow$ if $v_e \notin V(C)$ then $e_1, e_2 \notin E(C)$

and then $C \subset G$ which means v, w are in a cycle in G ✓

$\rightarrow$ if $v_e \in V(C)$ since $v, w \in G$, $v, w \neq v_e$

therefore, if write C as a walk

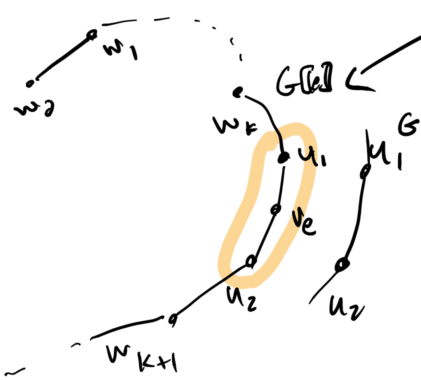
it has to look like in $G[\![e]\!]$

$$w_0 w_1 \cdots w_k \underbrace{u_1 v_e u_2}_{} w_{k+1} \cdots w_n w_0$$

$v \qquad X \qquad w$

$$w_0 w_1 \cdots w_k u_1 u_2 w_{k+1} \cdots w_n w_0$$

in G



Opposite direction

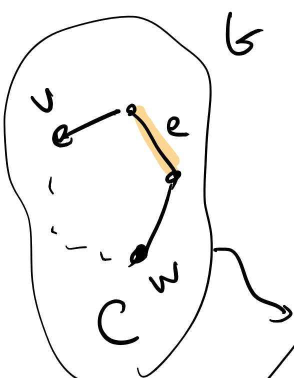
Suppose G is nonsep (every two vertices lie on a common cycle)

want to show same for $G[\![e]\!]$

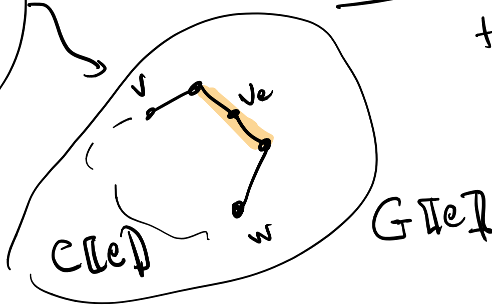
Choose $v, w \in G[\![e]\!]$ want to find a cycle in $G[\![e]\!]$ containing them.

<u>Case 1</u>: $v, w \neq v_e$ then, can regard $v, w \in G$ and can find a cycle $C \subset G$ containing v, w.

<u>Subcase a</u>: $e \notin E(C)$ can regard in this case, (as before) $C \subset G[\![e]\!]$ and so we're done



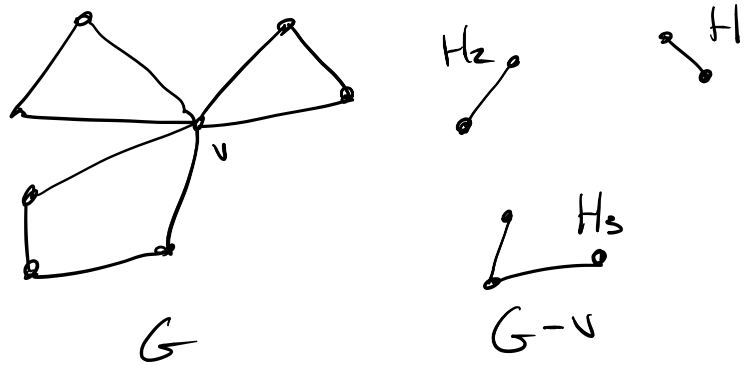
<u>Subcase b</u>: $e \in E(C)$ then $C[\![e]\!] \subset G[\![e]\!]$ is a cycle containing v & w



<u>Case 2</u>: if $v = v_e$ then finding a cycle in new graph $G[\![e]\!]$ containing w, v_e is like finding a cycle in original graph containing e & w.

Problem 2

Suppose v is a cut vertex in G and $H_1, \dots, H_k$ components of $G-v$. Show that if $C \subset G$ is a cycle containing v, then G can only intersect one of these components.



if C is a cycle, $v \in C$, then $C-v$ is connected, $C - v \subseteq G-v$

if $C \cap H_1 \neq \emptyset$ & $C \cap H_2 \neq \emptyset$ then

$(C-v) \cap H_1 \neq \emptyset$
$(C-v) \cap H_2 \neq \emptyset$

H_1 & H_2 are not connected to each other in $G-v$, so same in $C-v$