k-connected = k-vertex-connected = $(\kappa(G) \geq k)$

connected = 1-connected = $(\kappa(G) \geq 1)$

k-edge connected = $(\lambda(G) \geq k)$

3 connected $(\kappa(G) \geq 3)$

2 con... $(\kappa(G) \geq 2)$

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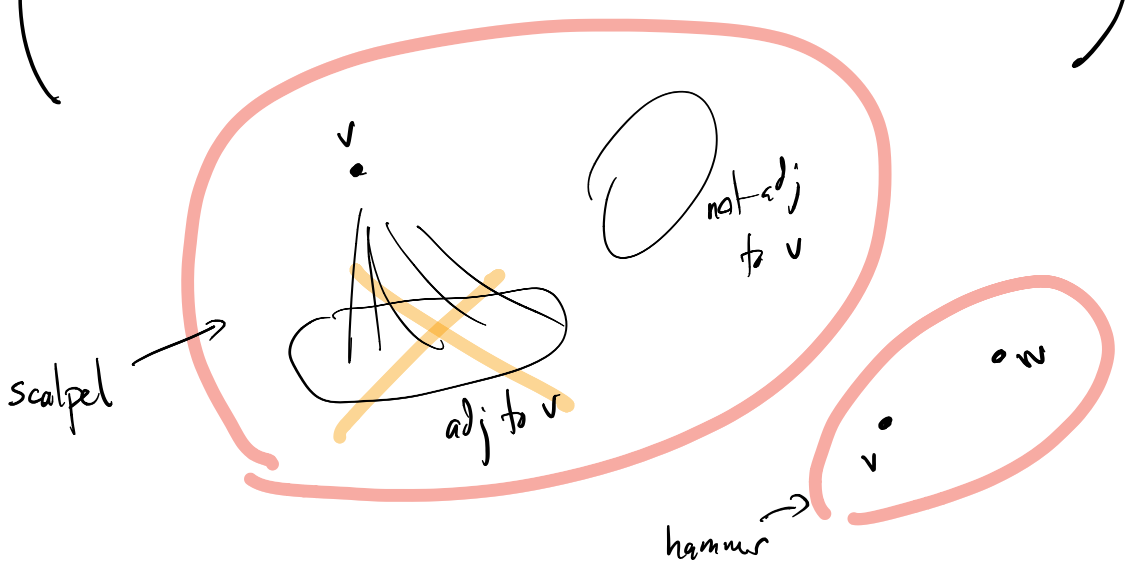
1. If G is not complete, show G has a vertex cut.

(G is not complete if and only if G has a vertex cut.)



4. Explain why $\delta(G) \geq \lambda(G)$

$\lambda(G)$ = minimum size of an edge cut.

if $\exists$ an edge cut of size k then $k \geq \lambda(G)$.

<u>Question</u>: $\exists$ an edge cut of size $\delta(G)$?

$\deg = \delta(G)$

remove all edges incident to v, $\deg(v) = \delta(G)$.

<u>In practice</u>: can "eyeball" $\kappa(G) \geq 1$ (connected)

$\kappa(G) \geq 2$

$\kappa(G) = 1 \Leftrightarrow \exists$ cut vertex

$\kappa(G) \geq 2 \Leftrightarrow$ nonseparable

$\kappa(G) = 4$ or 5? Hard.

value of $\kappa(G)$ computationally interesting.

values can vary widely over "similar pictures"

$$\boxed{\kappa(G) \leq \lambda(G) \leq \delta(G)}$$

$$\boxed{\begin{array}{l} \text{if } G = K_n \\ \kappa(G) = n-1 \end{array}}$$

7. If G is k-connected $k \geq 1$, $\forall v \in V(G)$ then $G - v$ is $(k-1)$-connected.

Pf: (Assume G is not complete)

G k-connected, have to remove at least k vertices to disconnect G

$(G - v) - S$ — if v was part of a min'l vertex cut, could remove $k-1$ more

?

Alternate viewpoint: $\boxed{\text{G is k-connected} \iff \text{G cannot be disconnected by removing k-1 vertices}}$ or trivial

if we remove v, no choice of $k-2$ more will make G disconnected.

[To show $G - v$ is $k-1$ connected, want to show that for any $S \subset V(G-v)$, $\# S = k-2$, then $\underbrace{(G-v) - S}_{G - (\underbrace{S \cup \{v\}}_{k-1 \text{ things}})}$ is connected and non trivial] or trivial

Case 2: G complete.

if G complete, say $G = K_n$ $(\kappa(G) = n-1)$

and G is k-connected

$\Leftrightarrow$

$\kappa(G) = n-1 \geq k$

$G - v \cong K_{n-1}$ $\kappa(G-v) = n-2 \geq k-1$

so $G-v$ is $k-1$ connected.

Alternative definition

Def $\kappa(G)$ = minimum size of a subset $S \subset V(G)$ such that $G-S$ is disconnected or trivial

$\text{trivial} = \bullet = K_1$

Note if we have G, $\#V(G) = n$.

if G is not complete then $\exists$ a vertex cut.

$\exists S \subset V(G)$ $G-S$ disconnected $\Rightarrow \#S \leq n-2$

to make $G-S$ trivial would require $\#S = n-1$

so S minimum size as above would be one making $G-S$ disconnected.

if G is complete, $G-S$ never disconnected, need $n-1$ vertices removed to make trivial

$\Rightarrow \kappa(G) = n-1$.