What's the point of Menger's Theorem? (~1930)

At the core:

two separate concepts

how "robust" is the connection between two vertices u, v?

how difficult is it to break the connection?

How many "distinct" ways can we get from u to v?

lots of distinct ways (internally disjoint paths) $\Rightarrow$ hard to break connection (u-v separating set)

Bird's eye view: when we have two qualitatively different descriptions of the same thing, typically we can discover *many* more facts which wouldn't have been obvious/easy otherwise.

Menger $\rightsquigarrow$

Theorem (Whitney) A nontrivial graph G is k-connected (for $k \geq 2$) if and only if for every $u, v \in V(G)$ there are at least k internally disjoint u-v paths.

Pf concept: if it takes at least the removal of k-vertices to separate any u,v, then there should be at least k int. disjoint u-v paths

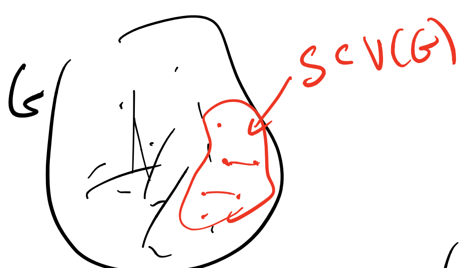
tricky parts: what if u,v are adjacent.

converse.

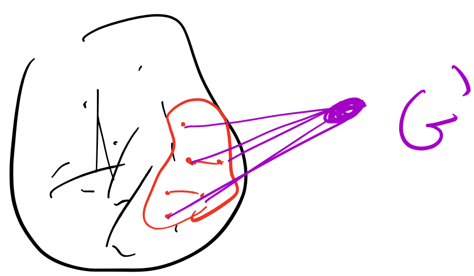
Abstractly useful to show k-connectedness

Suppose given that G is k-connected $\rightsquigarrow$ G'

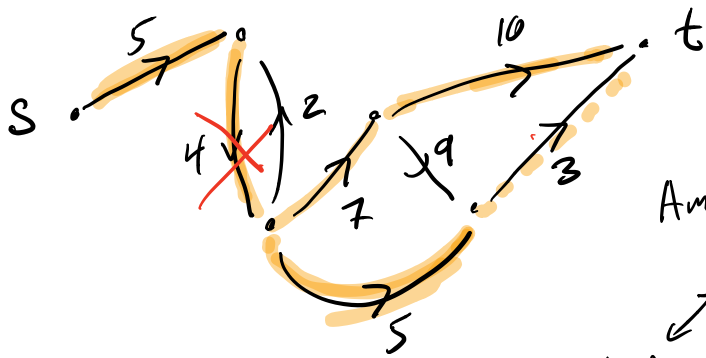
apply same procedure to all G



G' add a new vertex, and connect it to all vertices in S.



if G is k-connected, so is G'.



Amount can push through (4)

↔

\# indep paths

minimal amount of effort needed to cut s from t (4)

"min-cut/max-flow theorem"