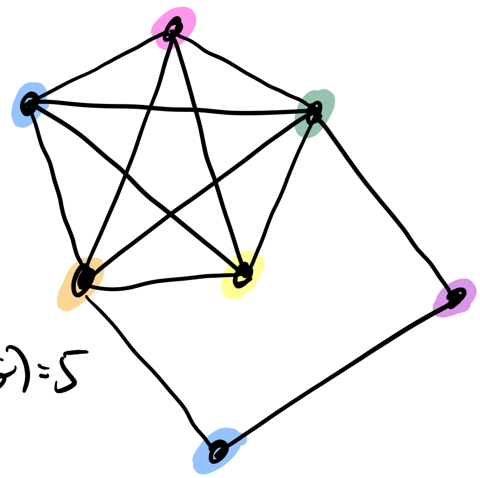
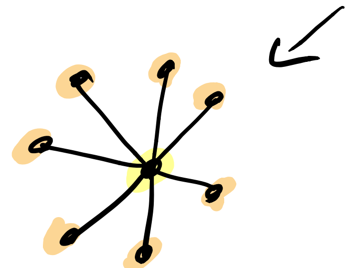
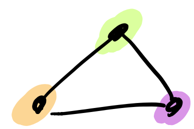


$\chi(G)=5$





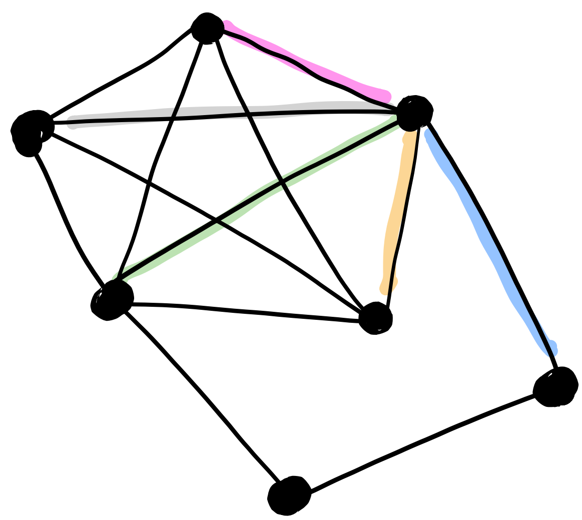
Brook's Theorem

If G is connected, not an odd cycle, not a complete graph, then $\chi(G) \leq \Delta(G)$

max degree of a vertex.

In general (thm 10.7)

$$\chi(G) \leq 1 + \Delta(G)$$



$$\Delta(G) + 1 \geq \chi'(G) \geq \Delta(G)$$

Vizing — Kőnig

<u>Q</u>: How hard is it to determine $\chi(G)$ & $\chi'(G)$

in general $\quad 2 \leq \chi(G) \leq \Delta(G)$ or $\Delta(G)+1$

G connected $\quad \Delta(G) \leq \chi'(G) \leq \Delta(G)+1$

χ — NP complete ← ∃ poly algorithms to get reasonable bounds on χ, good algorithms for various classes of graphs

χ' — NP hard

We will discuss some algorithms related to vertex colorings.

Contraction-deletion algorithm:

$$\text{runtime} \sim \left(\frac{1+\sqrt{5}}{2}\right)^{n+m}$$

n vertices

m edges

golden ratio