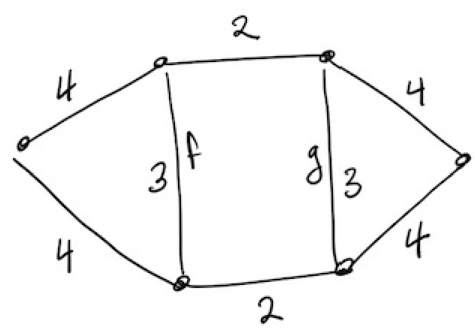
Graph Theory, Exam 1 Practice Sheet

1. Suppose G is a simple, connected graph and e is an edge in G. Show that there is a spanning tree of G containing e.

2. Recall that a edge e in a connected graph G is bridge if and only if $G - e$ is disconnected. Show that a connected graph G is a tree if and only if every edge in G is a bridge.

3. Show that if T is a tree and v is a vertex in T with $deg(v) = 3$, then T has at least 3 leaves.

4. Consider the graph G with edge weights shown below. Show that every minimal spanning tree in G contains one of the edges f or g, but not both.



5. Suppose G is composed of vertices $a_{i,j}$ for $i = 1, \ldots, 10$ and $j = 1, 2, 3$, and vertices b_1, b_2 with the following edges:

   - For $i \neq i'$, the vertices $a_{i,j}$ and $a_{i',j}$ are connected by an edge,
   - For all i, j, k, the vertices $a_{i,j}$ and b_k are connected by an edge,
   - The vertices b_1 and b_2 are connected by an edge,

   and no other edges are in the graph.

   Show that G cannot have a Hamiltonian cycle.

6. Call a graph G **double Hamiltonian** if there is a closed walk, starting at a vertex v, passing through no edge more then once, and passing through every vertex other than v exactly twice (and reaching v exactly 3 times).

   Give an example of a simple graph which is double Hamiltonian.

7. Is it possible to have a simple graph with vertices of degrees $5, 5, 5, 5, 4, 4$? How about $5, 5, 4, 4, 3, 2$?