Graph Theory Practice Sheet for Midterm 2

This sheet is not meant to be exhaustive, but rather as a supplement to the problems from the homework since the last exam.

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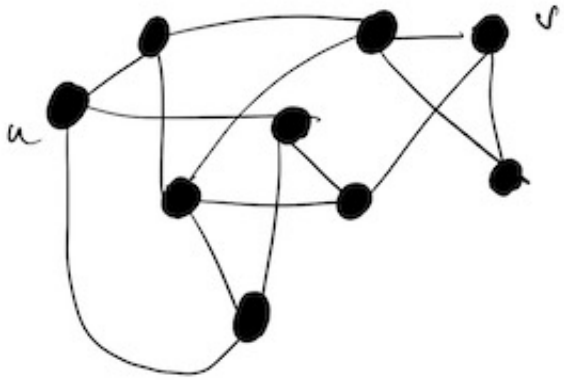
1. This is a problem in the direction of Vizing's Theorem. Show that for a graph G, you can always color it using at most $2\Delta - 1$ colors. As a hint, you should think about the simple vertex coloring algorithm and how it worked.
2. This problem is the direction of Brook's Theorem. Suppose that G is a graph, v is a cut vertex and $G_1, G_2, \ldots, G_k$ are the components of $G - v$. Show that if $\chi(G_i)$ is less than $\Delta(G)$ for each i, then we will also have $\chi(G) \leq \Delta$.
3. Can you draw a graph with $\chi(G) = 4$ and with the graph containing no triangles? If you can, do it. If not, say why not.
4. Draw a graph with $\kappa(G) = 2, \lambda(G) = 2$ and $\delta(G) = 3$ (or show no such graph exists).
5. In the graph shown below, exhibit a minimum $u - v$ vertex cut and a minimal $u - v$ vertex cut which isn't minimum. How can you tell that your minimum vertex cut is actually minimum?



6. Find all cut vertices and blocks in the graph below: