Last time(s)

Given a complex $\{C_n\}_{n\in\mathbb{Z}}, d,$
filtration $F_\bullet$ on $C_\bullet$ i.e. subcomplexes
$\{F_pC_n\}_{n\in\mathbb{Z}}$, defined a spectral sequence

$$E^0_{p,q} = F_pC_{p+q}/F_{p-1}C_{p+q} = gr^F_p C_{p+q}$$

with differential on 0^{th} page induced by

$$d: F_pC_{p+q} \to F_pC_{p+q-1}$$

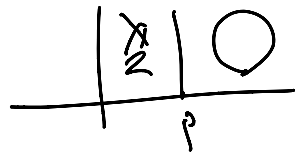
$$\text{w/ } E^0_{p,q} \Rightarrow H_{p+q}(C_\bullet)$$

$$\{E^r_{p,q}\}_{r\geq 0} \overset{r}{\Rightarrow} H_{p+q}$$

$$E^0_{p,q} = gr^F_p C_{p+q} \Rightarrow H_{p+q}(C_\bullet)$$

Recall: if $\{C_{p,q}\}$ double complex

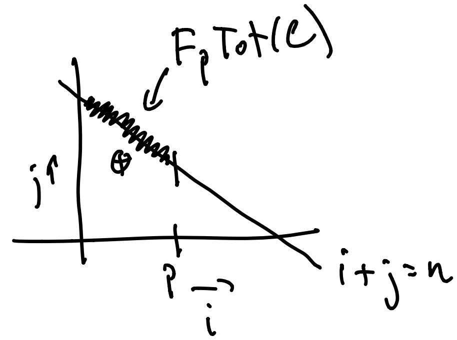
defn $\left({}^{I}\tau_{\le n} C_{\cdot\cdot}\right)_{p,q} = \begin{cases} C_{p,q} & \text{if } p \le n \\ 0 & \text{else} \end{cases}$



$$\left[\left({}^{II}\tau_{\le n} C\right)_{p,q} = \begin{cases} C_{p,q} & \text{if } q \le n \\ 0 & \text{else} \end{cases}\right]$$

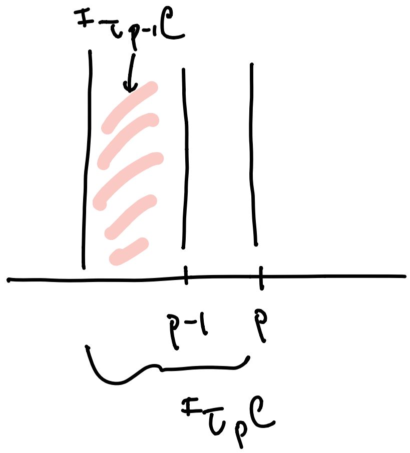
Gives a filtration on Total complex

$$F_p \mathrm{Tot}(C) = \mathrm{Tot}\left({}^{I}\tau_{\le p} C\right)$$



compute Spec seq:

$$E^0_{p,q} = F_p \mathrm{Tot}(C)_{p+q} \big/ F_{p-1} \mathrm{Tot}(C)_{p+q} = C_{p,q}$$



$$\frac{F_p \operatorname{Tot}(C)}{F_{p-1}\operatorname{Tot}(C)}$$

$$\operatorname{Tot}(\tau_{\le p} C)$$

$$= \operatorname{Tot}\Big(\bigoplus_{\substack{i \le p \\ j}} C_{ij}\Big)$$

$n = p+q$

deg n part $= \bigoplus_{\substack{i \le p \\ j = n-i}} C_{i,j}$

$$F_{p-1}\operatorname{Tot} C = \operatorname{Tot}\Big(\bigoplus_{\substack{i \le p-1 \\ j}} C_{ij}\Big)$$

deg n part: $\bigoplus_{\substack{i \le p-1 \\ j = n-i}} C_{i,j}$

quot (in deg n)

$C_{i,j}$

$i=p \quad j=n-p$

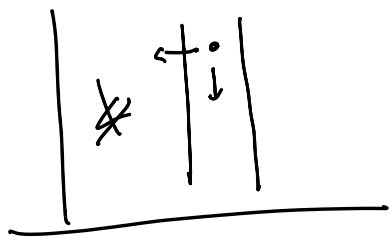
<u>Differential</u>: induced by

$$\begin{tikzcd} F_p D_{p+q} \arrow[r, "d"] & F_p D_{p+q} \\ F_{p-1} D_{p+q} \arrow[r, "d"] \arrow[u, phantom, "\cup"] & F_{p-1} D_{p+q} \arrow[u, phantom, "\cup"] \end{tikzcd}$$

differential

deg $n-1$ part of F_{p-1} filtr.

$$\begin{tikzcd}
\text{deg } n \text{ part} = \bigoplus\limits_{\substack{i\le p\\ j=n-i}} C_{i,j} \arrow[r, "\text{horiz}"] \arrow[dr, "\text{vert}"'] & \bigoplus\limits_{\substack{i\le p\\ j=n-i}} C_{i-1,j} = \bigoplus\limits_{\substack{i\le p-1\\ j=n-i-1}} C_{i,j} \\
& \bigoplus C_{i,j-1}
\end{tikzcd}$$



induced map $E^0_{p,q} \xrightarrow{d} E^0_{p,q-1}$ is d_{vert}

$C_{p,q} \longrightarrow C_{p,q-1}$

$E^1_{p,q} = H_q(C_{p,*})$

For E^2

induced diff.s on $E^1_{p,q}$ pge i

given $c \in H_q(C_{p,*})$ rep. by $\tilde{c} \in C_{p,q}$

w/ $d\tilde{c} \in C_{p-1,q}$ i.e. $d_v c = 0$

$\overset{\shortparallel}{d_h \tilde{c}}$ (sme

$d = d_v + d_h$ $\quad \tilde{c}$ is a cycle for d_v

($\tilde{c}$ is a cycle on induced $F_pD/F_{p-1} \to F_p/F_{p-1}$)

$$\boxed{\text{Exercise } 5.1.2}$$