

Brief Recollection

given $\mathcal{F}: A \rightarrow B$ right exact

$A \in \text{Ch}(A)$ define (when we can)

$$L_n^{\mathcal{F}}(A) = H_n(\mathcal{F} \text{Tot}^{\oplus} P_{..})$$

where $P_{..} \rightarrow A_{..}$ is an EM resolution

in general, if $C_{..}$ is a double complex

we have spectral sequences

$$\{ I E_{p,q}^* \}$$

filtration by
columns (p)

$$\{ II E_{p,q}^* \}$$

filtration by
rows (p!)

$$II E_{p,q}^2 = H_p^v H_q^h(C) \begin{matrix} \uparrow \\ C_{q,p} \end{matrix}$$

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converge to $H_{p+q}(\text{Tot}^{\oplus} C)$ when
they are "suff. bounded."

For computing $L_n \mathcal{F}(A)$ w/ $P_{..} \rightarrow A$.

$$H_0^v(\mathcal{F}P_{..}) = L_0 \mathcal{F}(A_*)$$

$${}^I E_{p,q}^2 = H_p(L_0 \mathcal{F}(A))$$

$$H_0^h(\mathcal{F}P_{..}) = \mathcal{F}P_{0*}^H$$

res. of $H_0(A)$

$$H_p^v(H_0^u \mathcal{F}P) = L_p \mathcal{F}(H_0 A)$$

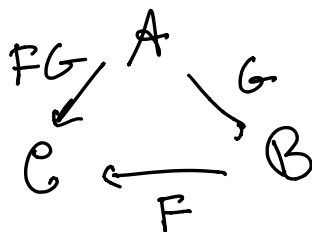
$$\text{"}$$

$${}^II E_{p,q}^2$$

$$L_p \mathcal{F}(H_0 A) \Rightarrow L_{p+0} \mathcal{F}(A) \text{ always}$$

$$H_p(L_0 \mathcal{F}(A)) \Rightarrow L_{p+0} \mathcal{F}(A) \text{ if } A \in \mathcal{C}h_{\geq 0}$$

Grothendieck Spectral sequence



F, G left exact
 enough injectives in A, B
 and G takes injectives
 to F -acyclics

Reminder: $A \in \mathcal{B}$ is F -acyclic if $R^p F(A) = 0$
if $p > 0$

for any $A \in \mathcal{A}$,
Then we have a convergent spectral seq.

$$E_2^{p,q} = (R^p F) \left((R^q G)(A) \right) \Rightarrow R^{p+q}(FG)(A)$$

PF: Construction:

given $A \in \mathcal{A}$, choose inj. res $A \rightarrow I$.

apply G to get $GA \rightarrow GI$.

$$R^n F(GI)$$

$$\text{have } \begin{matrix} I \\ E_2^{p,q} \end{matrix} \Rightarrow R^n F(GI)$$

$$H^p \left((R^q F)(GI) \right) \quad GI \text{ is acyclic}$$

$$(R^q F)(GI) = 0$$

if $q > 0$

$$\Rightarrow R^n F(GI) = H^n \left((R^0 F)(GI) \right)$$

$$H^n \left((FG)(I) \right) = R^n (FG)(A)$$

$$\mathbb{H} E_2^{p, q} = R^p F \left(\underbrace{H^q(GI)}_{\cong R^q G(A)} \right) = R^p F (R^q G(A))$$

$$\mathbb{H} E_2^{p, q} = R^p F (R^q G(A)) \Rightarrow R^n F (GI) \\ \cong R^n (FG)(A)$$