In videas, showed that if we have an &-function of Tizs: A -> B > d. each Ti, iso

The coefficients, then {T:355 universal,

and so all the "satellite function" Ti, iso

me uniquely deturined by To.

Do they exist? Constitues via devined fuctors.

Basic procedure?

Cren our right exact finch F, in object A,

Choose a complex >P2 > P, -> Po -> A -> O

Choose a complex -> P2 -> P, -> Po -> A -> O

exad.

gray a gisam P. -> A

where each Pi is a projecte object of A.

There is a projecte object of A.

There is a projecte object of A.

Then we done $F_{i}(A) = H_{i}(FP.)$ hambur $FP_{i+1} \to FP_{i-1}$

Step 1' Prejecte objects Lemma let A be an Alelian category, Lemma Let /

Po A then TFAE:

1) Hom (P, -) is right exact

2) Every SES 0 - A - B - P - > 0 splits

2) Any epic map A - P splits

4) gren any drayram P exact, 7 a

B -> C -> 0 map P -> B

s.t. digram

camustes Pf (3=)4) Suppose 3 holds, consider Defre $A \times_B P = kr \left(A \times P g \frac{\pi_1 - f \pi_2}{2} B \right)$ note, by constitut, have a commideyram

AxBP TTZ P

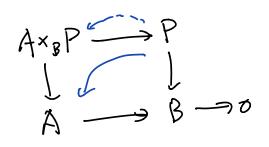
Jf

Claim: TTZ Tseptc

A & B > 0

let X&A, let x&P(X). Set x&B(X)
its image. since g is epro, I y&A(X)
mappy to x. Then (x,-y)&(A xBP)(X)
and $\pi_2(x,-y)=X$.

By hypothesis, Ax&P -> P has a splitto



Det P& As is called projecte if the equis conditions of the Lemma hold.

Main point, additue function always presence split exact sequences. If Pisprojecte, gren av. exact Fo=F suppase ue can find G ->> P w/ F, 6=0 On Knon Pno splits ⇒LES h{F;} is actually a cures of F; K @ F; P = F; G Learner: additue functions present sophit exact acqueres. Pf. A=B1@B2 nears re hare A TiBi $5.1- Tili=id_{3}$ 2i $2iT_1+2iT_2=id_A$

Projecte object Det an As. cat A has enough projecties st HAEA, 3P -> A ul Pprojecte. ex' Mode has enough projectes. Free madules are projectue! Home (R, M)=M easy to check: 10 1s of projectes one projecte. Mode has enoth projected becaux my module has a generally set. R" ->> M Summand of projectives are projecte N -> N-> 0

Propi An Romande Mis proj.

Pti et Mis proj, consider a surjection
RF -> M, which splits. D.

Exi if R = PIP, M l.g. proj > free.

if R is division, M proj > free.

if R is a comm. lowing. proj > free.

Q = 21/62 P = (21/62)/(221/62) = 21/2211 $21/32 \times 24/22$ Is pay, not free.

Z/IJ-5] has nontrée parjectes.

Det A resolution frandspect AEX is an complex P. whamy Po-A. s.t. - P2->P,->P0->A->0 it is a projecte resolution; tall pi's are projectre. Two handy lummes: lem1 if A & B in & and P. Q. are proj. res'ns & A + B resp. then 3 f.: P. -> Q. s.t. P. -> A If I Lo [f commes

and further, to 13 unique up to homotopy equivalence.

lend if $0 \rightarrow A' \rightarrow A'' \rightarrow 0$ it sess and P' of P'' are proj. resolutions . [A', A'] then can find a projeres Post to we each

Pin Pi & Pi, grey a SES of complexes

O - Pi - Pos - Pi - > O (split frenchi)