

MATH 1610, Honors Calculus 1, Fall 2026, Exam 1

Instructor: Danny Krashen

Name _____

(Please write your full name, as neatly as possible)

Instructions:

Students are not allowed to use calculators and must work alone.
The only materials allowed are: writing instruments, blank scratch paper, and one single page of notes,
handwritten or printed (standard 8.5 x 11 or A4 sheet, both sides).

Please read and sign the academic honesty statement before the start of the exam.

This exam has 5 questions.

University of Pennsylvania Code of Academic Integrity

Since the University is an academic community, its fundamental purpose is the pursuit of knowledge. Essential to the success of this educational mission is a commitment to the principles of academic integrity. Every member of the University community is responsible for upholding the highest standards of honesty at all times. Students, as members of the community, are also responsible for adhering to the principles and spirit of the following Code of Academic Integrity.

Academic dishonesty are those activities that have the effect or intention of interfering with education, pursuit of knowledge, or fair evaluation of a student's performance are prohibited. Examples of such activities include but are not limited to the following definitions:

- *Cheating*: using or attempting to use unauthorized assistance, material, or study aids in examinations or other academic work or preventing, or attempting to prevent, another from using authorized assistance, material, or study aids. Example: using a cheat sheet¹ in a quiz or exam, altering a graded exam and resubmitting it for a better grade, etc.
- *Plagiarism*: using the ideas, data, or language of another without specific or proper acknowledgment. Example: copying another person's paper, article, or computer work and submitting it for an assignment, cloning someone else's ideas without attribution, failing to use quotation marks where appropriate, etc.
- *Fabrication*: submitting contrived or altered information in any academic exercise. Example: making up data for an experiment, fudging data, citing nonexistent articles, contriving sources, etc.
- *Multiple submission*: submitting, without prior permission, any work submitted to fulfill another academic requirement.
- *Misrepresentation of academic records*: misrepresenting or tampering with or attempting to tamper with any portion of a student's transcripts or academic record, either before or after coming to the University of Pennsylvania. Example: forging a change of grade slip, tampering with computer records, falsifying academic information on one's resume, etc.
- *Facilitating academic dishonesty*: knowingly helping or attempting to help another violate any provision of the Code. Example: working together on a take-home exam, etc.
- *Unfair advantage*: attempting to gain unauthorized advantage over fellow students in an academic exercise. Example: gaining or providing unauthorized access to examination materials, obstructing or interfering with another student's efforts in an academic exercise, lying about a need for an extension for an exam or paper, continuing to write even when time is up during an exam, destroying or keeping library materials for one's own use, etc.

I agree to uphold the above code of academic integrity in this course and on this examination.

Signature _____

¹you are allowed a single "cheat sheet" on this exam though!

Reminders

A **field** is a set F with two operations $+$ and $\cdot$ and two distinguished elements $0 \neq 1$, such that:

- $+$ is associative and commutative, $a + 0 = a$ for all a , and every $a \in F$ has an additive inverse $-a$ with $a + (-a) = 0$;
- $\cdot$ is associative and commutative, $a \cdot 1 = a$ for all a , and every $a \neq 0$ has a multiplicative inverse a^{-1} with $a \cdot a^{-1} = 1$;
- multiplication distributes over addition: $a(b + c) = ab + ac$.

An **ordered field** is a field F together with a subset $F^+ \subseteq F$, the positive elements, such that:

- F^+ is closed under addition and under multiplication;
- for every $a \in F$, exactly one of the following holds: $a \in F^+$, $-a \in F^+$, $a = 0$.

We then write $a < b$ to mean $b - a \in F^+$.

1. Let $f : X \rightarrow Y$ be a function, and let $C, D \subseteq Y$. Recall that $f^{-1}(C) = \{x \in X \mid f(x) \in C\}$. Show that

$$f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D).$$

2. Suppose $A' \subseteq A$ and $B' \subseteq B$ are nonempty subsets of $\mathbb{R}$, and suppose that $a < b$ for every $a \in A$ and every $b \in B$.
- (a) Show that $\sup A \leq \inf B$. Is it possible that these are equal? Either show this cannot happen, or give an example to show it can.

(b) Show that $\sup A' \leq \sup A$ and $\inf B' \geq \inf B$.

(c) Show that if $\sup A' = \inf B'$ then $\sup A = \inf B$.

3. Use the definition of a limit to show that $\lim_{x \rightarrow 4} 1/x = 1/4$.

4. Suppose F is an ordered field and $a, b \in F$ are nonzero elements with $a < b$ and $1/a < 1/b$. Show that ab is negative.

5. Recall that for a bounded function $f : [a, b] \rightarrow \mathbb{R}$, the lower integral of f is

$$L\int_a^b f(x) \, dx = \sup \left\{ \int_a^b s(x) \, dx \mid s \text{ a step function with } s(x) \leq f(x) \text{ for all } x \in [a, b] \right\}.$$

Suppose $f, g : [a, b] \rightarrow \mathbb{R}$ are bounded functions with $f(x) \leq g(x)$ for all $x \in [a, b]$. Show that

$$L\int_a^b f(x) \, dx \leq L\int_a^b g(x) \, dx.$$