# Math 1610, Honors Calculus 1, Fall 2026, Homework 1 — Solutions

**Please note.** These solutions were generated agentically (by an AI assistant). They have been looked over, but they may still contain errors. If you find a mistake, or anything that is unclear or could be better explained, please let us know — we would be glad to hear about it.

1. Prove, using the definitions of intervals and set equality, that $(-4,5] \cap [1,7) = [1,5]$.

**Solution:** We show the two inclusions.

$\subseteq$: Suppose $x \in (-4,5] \cap [1,7)$. Then $x \in (-4,5]$, so $-4 < x \leq 5$, and $x \in [1,7)$, so $1 \leq x < 7$. The second of these gives $1 \leq x$ and the first gives $x \leq 5$, so $1 \leq x \leq 5$, which is to say $x \in [1,5]$.

$\supseteq$: Suppose $x \in [1,5]$, so that $1 \leq x \leq 5$. Since $-4 < 1$ and $1 \leq x$ we have $-4 < x$, and we are given $x \leq 5$; hence $x \in (-4,5]$. Since $1 \leq x$ and $x \leq 5 < 7$ we have $1 \leq x$ and $x < 7$; hence $x \in [1,7)$. Being in both sets, $x \in (-4,5] \cap [1,7)$.

Since each set is contained in the other, they are equal.

2. Prove that if $A \subseteq X$ and $B \subseteq X$ then $A \cup B \subseteq X$.

**Solution:** If $x \in A \cup B$ then either $x \in A$ or $x \in B$. But since $A \subseteq X$ and $B \subseteq X$, if $x \in A$ then $x \in X$, and if $x \in B$ then $x \in X$. So $x \in X$ in either case.

3. Suppose $X$ and $Y$ are sets and $f : X \to Y$ is a function.

   (a) If $A$ and $B$ are subsets of $X$, is it always true that $f(A) \cup f(B) = f(A \cup B)$?

   **Solution:** Yes, this always holds.

   $\subseteq$: Let $y \in f(A) \cup f(B)$, so $y \in f(A)$ or $y \in f(B)$. If $y \in f(A)$, there is some $x \in A$ with $f(x) = y$. Since $A \subseteq A \cup B$ we have $x \in A \cup B$, and therefore $y \in f(A \cup B)$. The case $y \in f(B)$ is identical.

   $\supseteq$: Let $y \in f(A \cup B)$. Then $y = f(x)$ for some $x \in A \cup B$, so $x \in A$ or $x \in B$. If $x \in A$ then $y = f(x) \in f(A)$; if $x \in B$ then $y = f(x) \in f(B)$. Either way $y \in f(A) \cup f(B)$.

   (b) If $A$ and $B$ are subsets of $X$, is it always true that $f(A) \cap f(B) = f(A \cap B)$?

   **Solution:** No. Let $X = \{1,2\}$, let $Y = \{0\}$, and let $f : X \to Y$ be the (only) function, so $f(1) = f(2) = 0$. Take $A = \{1\}$ and $B = \{2\}$.
   Then $A \cap B = \varnothing$, so $f(A \cap B) = \varnothing$. On the other hand $f(A) = \{0\}$ and $f(B) = \{0\}$, so

$$f(A) \cap f(B) = \{0\} \neq \varnothing = f(A \cap B).$$