Math 1610, Honors Calculus 1, Fall 2026, Homework 1

1. We define interval notation for subsets of the real numbers as follows. Recall that $\mathbb{R}$ denotes the set of real numbers.

   For real numbers $a < b$, we define:

   $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$

   $(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

   $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

   $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

   Prove, using the definitions of intervals and set equality that $(-4, 5] \cap [1, 7) = [1, 5]$.

2. Prove that if $A \subseteq X$ and $B \subseteq X$ then $A \cup B \subseteq X$.

3. Suppose X and Y are sets and $f : X \to Y$ is a function from X to Y.

   (a) If A and B are subsets of X, is it always true that $f(A) \cup f(B) = f(A \cup B)$? Either prove that this is always true, or illustrate an example showing that it doesn't always hold.

   (b) If A and B are subsets of X, is it always true that $f(A) \cap f(B) = f(A \cap B)$? Either prove that this is always true, or illustrate an example showing that it doesn't always hold.