

Math 1610, Honors Calculus 1, Fall 2026, Homework 5

1. Let $S \subseteq \mathbb{R}$, and let $f, g : S \rightarrow \mathbb{R}$ be strictly increasing.

(a) Show that $f + g$ is strictly increasing.

(b) Is fg always strictly increasing? Either prove it, or give an example showing it can fail.

- (c) Suppose $T \subseteq \mathbb{R}$, that $h : S \rightarrow T$ is strictly increasing, and that h has an inverse function $h^{-1} : T \rightarrow S$. Show that h^{-1} is also strictly increasing.

2. Show that the equation

$$x^5 + x = 1$$

has exactly one solution in $[0, 1]$.

there are two things to do here: show that you have a solution, and show that you can't have more than one. and keep in mind, you do not need derivatives for this – we don't have them yet!

3. Suppose $f, g : [0, 1] \rightarrow [0, 1]$ are both continuous, and that

$$f(0) \leq g(0) \quad \text{and} \quad f(1) \geq g(1).$$

Show that there is some $c \in [0, 1]$ with

$$f(c) = g(c).$$

(In words: f starts out no higher than g and ends up no lower, so somewhere along the way the two graphs must meet.)

4. In lecture we showed that a continuous function on a closed interval $[a, b]$ is bounded, and attains its supremum. This problem is about why those hypotheses are there. In each case, give an example and briefly say which hypothesis of the theorem fails.

(a) A continuous function on the open interval $(0, 1)$ which is not bounded.

(b) A continuous, bounded function on $(0, 1)$ which does not attain its supremum.

(c) A function on the closed interval $[0, 1]$ which is not bounded.