Math 1610, Honors Calculus 1, Fall 2026, Homework 8

Throughout, curves $\vec{r}\colon [a,b] \to \mathbb{R}^n$ have continuous derivatives, and vector fields are continuous.

1. (Running a curve backwards.) Let $\vec{r}\colon [a,b] \to \mathbb{R}^n$ be a curve and $\vec{F}$ a vector field on $\mathbb{R}^n$. The curve $\overline{r}(t) = \vec{r}(a+b-t)$, $t \in [a,b]$, traces the same path from $\vec{r}(b)$ to $\vec{r}(a)$. Show that

$$\int_a^b \vec{F}\big(\overline{r}(t)\big) \cdot \overline{r}\,'(t)\,dt = -\int_a^b \vec{F}\big(\vec{r}(t)\big) \cdot \vec{r}\,'(t)\,dt.$$

2. (Using a potential.) Let

$$\vec{F}(x, y, z) = \big(2xy + z^2,\ x^2,\ 2xz\big).$$

(a) Find a function $G\colon \mathbb{R}^3 \to \mathbb{R}$ with $\nabla G = \vec{F}$. *(Which functions have $\partial G/\partial x = 2xy + z^2$?)*

(b) Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is traced by

$$\vec{r}(t) = \big(1 + 7t - 7t^2,\ t^3,\ t^2\big), \qquad t \in [0, 1].$$

(c) For a number k, let $\vec{F}_k(x, y, z) = \big(2xy + z^2,\ x^2,\ kxz\big)$. For which k is $\vec{F}_k$ the gradient of some function $G\colon \mathbb{R}^3 \to \mathbb{R}$? Prove your answer.

3. (Not a gradient.) Let $\vec{F}(x, y) = \big(y^2,\ 3xy\big)$.

   (a) Compute the line integrals of $\vec{F}$ from $(0, 0)$ to $(1, 1)$ along $\vec{r}_1(t) = (t,\ t)$ and along $\vec{r}_2(t) = (t,\ t^2)$, $t \in [0, 1]$.

(b) Show that $\vec{F}$ is not the gradient of any function $G\colon \mathbb{R}^2 \to \mathbb{R}$.