Math 3120, Linear Algebra, Spring 2026, Homework 1

1. (for credit)

   Use elementary row operations to find the inverse of the matrix

   $$\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

   (Show your row reduction steps clearly)

2. (for credit)

   Use the previous answer to write a solution to

   $$\begin{aligned} x + 3y &= 1 \\ 2x + y &= 7 \end{aligned}$$

3. (for completion)

   Suppose we have a system of 4 equations in 5 unknowns. Use the definition of rank and row echelon form to show that if the rank of the system is 4, there is always at least one solution.

4. (for completion)

   Suppose we have a system of n equations in n unknowns. Write a short paragraph to explain why, if the rank of the system is n, one can use elementary matrices and row operations to show that the system has a unique solution.

5. (for credit)

   Consider row vectors R_1, R_2 both of length 12. Let R be the 2×12 matrix with R_1 on the top and R_2 on the bottom. Describe a matrix T (by explicitly giving its size and entries) such that TR is the 1×12 (row) matrix $R_1 - R_2$.

6. (for completion)

   Suppose R is a 1×5 matrix (with a single row of length 5) and C is a 15×1 matrix (with a single column of length 15). Let A be the 15×5 matrix CR. Suppose that both R and C are nonzero (i.e. each has at least one nonzero entry). Explain why the rank of CR is exactly 1.

7. (bonus)

   Show that if A is an $m \times n$ matrix with rank exactly 1, then it must be of the form CR for C a $m \times 1$ matrix and R a $1 \times n$ matrix

8. (bonus)

   How might you do a similar procedure to write a matrix of rank 2? Justify your thinking!