

## MATH 3120 — Homework 2

### 1. Linear transformations and matrices (11 points).

Consider the following linear transformations from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ :

- $T$  takes a vector and rotates it  $90^\circ$  counterclockwise.
- $S$  takes a vector  $(a, b)$  and swaps the entries, giving  $(b, a)$ .
- $U$  takes a vector  $(a, b)$  and subtracts the second entry from the first, giving  $(a - b, b)$ .

1. (2 points) What is the matrix representing  $S$ ?

$$\begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix}$$

2. (2 points) What is the matrix representing  $T$ ?

$$\begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix}$$

3. (2 points) What is the matrix representing  $U$ ?

$$\begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix}$$

4. (2 points) What is the matrix representing  $T^{-1}$ ?

$$\begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix}$$

5. (3 points) Suppose  $A$  is the matrix given by the product  $UTS$ . Find the matrix  $A^{-1}$ .

$$\begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix}$$

### 2. Interpreting a matrix column (3 points).

Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear transformation with matrix  $A$ .

What does the first column of  $A$  represent? Circle one.

- (a) the image of  $(0, 0, 1)$  under  $T$
- (b) the image of  $(1, 0, 0)$  under  $T$
- (c) the  $x$ -coordinate of the output of  $T$
- (d) the determinant of  $T$

3. **Matrix-action multiple choice (3 points).**

Suppose

$$A = \begin{pmatrix} 2 & 5 & -1 \\ 3 & 4 & 7 \end{pmatrix}$$

is the matrix of a linear transformation

$$T : \mathbb{R}^3 \rightarrow \mathbb{R}^2.$$

What is  $T(0, 1, 0)$ ? Circle one.

- (a)  $(2, 3)$
- (b)  $(5, 4)$
- (c)  $(-1, 7)$
- (d)  $(3, 4)$

4. **Squaring a transformation (3 points).**

Let  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation that swaps the entries of a vector:

$$S(a, b) = (b, a).$$

What transformation does  $S^2$  represent? Circle one.

- (a) reflection across the  $x$ -axis
- (b) rotation by  $90^\circ$
- (c) the identity transformation
- (d) projection onto the line  $y = x$