

Math 3120, Linear Algebra, Spring 2026, Homework 6

For full credit, you must show your work and communicate what you are doing!

1. Consider the vector space P_2 of polynomials of degree at most 2, and consider the map $T : P_2 \rightarrow \mathbb{R}^3$ defined by $T(f) = (f(-1), f(0), f(1))$.
 - (a) Consider the basis for $\mathcal{B} = (1, x, x^2)$ for P_2 , and let $\mathcal{E}$ be the standard basis for $\mathbb{R}^3$. What is the matrix ${}_{\mathcal{E}}T_{\mathcal{B}}$ for T with respect to these bases?
 - (b) What is the rank of T (i.e., the dimension of its image)? What is its nullity (i.e, the dimension of its kernel)? Is T invertible?

2. Consider the vector space P_2 of polynomials of degree at most 2, and consider the map $T : P_2 \rightarrow \mathbb{R}^3$ defined by $T(f) = (f'(-1), f'(0), f'(1))$.

(a) Consider the basis for $\mathcal{B} = (1, x, x^2)$ for P_2 , and let $\mathcal{E}$ be the standard basis for $\mathbb{R}^3$. What is the matrix ${}_{\mathcal{E}}T_{\mathcal{B}}$ for T with respect to these bases?

(b) Now consider the basis $\mathcal{F}$ for $\mathbb{R}^3$, given by $\mathcal{F} = ((1, 1, 1), (1, 1, 0), (1, 0, 0))$. Find the matrix ${}_{\mathcal{F}}T_{\mathcal{B}}$.

(c) What is the rank of T (i.e., the dimension of its image)? What is its nullity (i.e., the dimension of its kernel)? Is T invertible?

3. Consider the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) Exhibit how to perform a sequence of row and column operations to transform A into a new matrix B such that both B and B^T are in reduced row-echelon form.

(b) What is the rank of A ? What is the nullity of A ? Justify your answer.

- (c) Let T be the linear transformation $\mathbb{R}^3 \rightarrow \mathbb{R}^2$ given by the matrix A . Show that you can find a basis $\mathcal{B}$ for $\mathbb{R}^3$ and a basis $\mathcal{C}$ for $\mathbb{R}^2$ so that ${}_CT_B = B$.