Canvas:

- Homework via gradescope
- Announcements, emails.
- In class polling
- grade information.

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Research in abstract algebra.

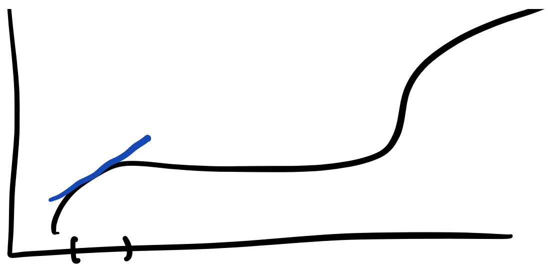
Basic tension in life:

- Linear equations are easy to solve $3x = 8$

  $2x - y = 4 \quad x + y = 9$

- Most descriptions of things in real life are very <u>non linear</u> - or worse - have no convenient understood explicit descriptions

Miracle: even though almost nothing is linear, things are often well approximated by linear systems in practical applications.



Starting points

Solving systems of linear equations

warm up

$$\begin{aligned} 4x + y &= 10 \\ + \quad x - y &= 5 \\ \hline 5x &= 15 \\ x &= 3 \end{aligned}$$

$\longrightarrow 3 - y = 5$

$y = 3 - 5 = -2$

$$\begin{aligned} 2x - y + z &= 4 \\ x + y - z &= -1 \\ x + 2y + 1 &= 3 \end{aligned}$$

$\overset{+}{\Rightarrow}\quad 3x = 3 \qquad x = 1$

$x = 1$

$$\begin{aligned} 2 - y + z &= 4 \\ 1 + y - z &= -1 \\ 1 + 2y + 1 &= 3 \end{aligned}$$

$3x = 3$

$x = 1$

"augmented matrix"

$$+\left[\begin{array}{ccc|c} 2 & -1 & 1 & 4 \\ 1 & 1 & -1 & -1 \\ 1 & 2 & 1 & 3 \end{array}\right]$$

$$\frac{1}{3}\left[\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 1 & 1 & -1 & -1 \\ 1 & 2 & 1 & 3 \end{array}\right]$$

$$\begin{array}{c} - \\ \\ - \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 2 & 1 & 3 \end{array}\right]$$

$y - z = -2$

$2y + z = 2$

$$\begin{array}{c} \\ \\ + \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$3y = 0$

$$\begin{array}{c} \\ \frac{1}{3} \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$$\begin{array}{c} \\ -2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 2 \end{array}\right]$$

$x = 1$

$y = 0$

$z = 2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \cancel{-2\cdot 0} \end{array}\right]$$

Another perspective:

Matrix = verb.

$2x - y + z = 4$

$x + y - z = -1$

$x + 2y + 1 = 3$

general $\rightsquigarrow$

$2x - y + z = a$

$x + y - z = b$

$x + 2y + 1 = c$

System describes a special function T taking inputs (x,y,z) to outputs $(a,b,c) = T(x,y,z)$

two ways to read this

- given a,b,c solve for x,y,z
- given x,y,z, procedure to produce a,b,c.

"inverting T"

Location: Sections 1.1, 1.2 in textbook

Language:

Definition: An equation of the form

$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$ is called a linear eqn.

a_i's = coefficients (real numbers)

x_i's = variables / indeterminants.

b = constant term

$3x + y + z = 7$ coeffs $3,1,1$

vars x,y,z

const 7

Def: A system of lnr equations is a list of lnr equs.

$$a_1x_1+a_2x_2+\dots+a_nx_n=b_1$$
$$c_1x_1+c_2x_2+\dots+c_nx_n=b_2$$
$$d_1x_1+d_2x_2+\dots+d_nx_n=b_3$$

$$a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+\dots\qquad=b_2$$
$$\vdots$$

Def: A solution to a system of lnr equations

$$a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+\dots\qquad=b_2$$
$$\vdots$$

is a sequence of real numbers $s_1, s_2, \dots, s_n$ such that all equations hold when we substitute

$$x_1 \rightsquigarrow s_1$$
$$x_2 \rightsquigarrow s_2$$
$$\vdots$$
$$x_n \rightsquigarrow s_n$$

Def: A system of equs is called consistent if it has at least 1 solution, otherwise it's called inconsistent.

$\left.\begin{array}{l} x=1 \\ x=2 \end{array}\right\}$ inconsistent $x+y=0$ $\}$ consistent

Typical warm up task:

Given a system, determine if consistent & if so describe all solutions.

ex: $x=3$

ex: $x-2y-z=13$

$x = 13+2y+z$

$x=13 \quad y=0 \quad z=0$

$x=14 \quad y=0 \quad z=1$

"Parametric solution"

$s,t \rightsquigarrow \begin{array}{l} y=s \\ z=t \\ x=13+2s+t \end{array}$

s,t "free parameters"

"The ~~Algorithm~~"
Moves

"Elementary operations"

Given a system of eqns

i) interchange eqns

ii) multiply an eqn by a nonzero number.

iii) add a multiple of one row to another.

Two basic observations:

- when we do these, result is always implied by the original system.
- these are reversable.