

Note -- this wasn't really a lecture, we just worked on a worksheet during the period!

$$1. \quad v_0 = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v_1 = \frac{\sqrt{2}}{2} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} .7 \\ .7 \end{bmatrix} \quad \begin{bmatrix} -.7 \\ .7 \end{bmatrix}$$

$$2. \quad A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{bmatrix} \quad \text{e-vectors } w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ eval. at } 2$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2a \\ \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix} \end{bmatrix}$$

$$Aw = 2w$$

$$\text{e-vectors } \begin{bmatrix} 0 \\ .7 \\ .7 \end{bmatrix} \quad \begin{bmatrix} 0 \\ -.7 \\ .7 \end{bmatrix}$$

$$3 \quad a) \quad \text{oracle: } (0.54, -0.55, -0.63)$$

$$b) \quad \text{basis } \left\{ \begin{pmatrix} 0 \\ 0.63 \\ -0.55 \end{pmatrix}, \begin{pmatrix} 0.63 \\ 0 \\ 0.54 \end{pmatrix} \right\} \text{ span } W$$

$$c) \quad \text{Find an orthonormal basis of } W.$$

↙ rescale it.
↘ add / subtract some of them to make $\perp$.

$$(0, .75, -.66) = v_1$$

$$(0.84, 0.36, 0.4) = v_2 = \text{proj}_{v_1} + (0.63, 0, 0.54) \perp \text{ to } v_1$$

$$Av_1 = \begin{pmatrix} 0.57 \\ -1.23 \\ 1.58 \end{pmatrix} = c_1 v_1 + c_2 v_2$$

$$c_2 \cdot 0.84 = 0.57$$

$$c_2 = 0.57 / 0.84$$

$$-1.23 = c_1(.75) + c_2(0.36)$$

↑
0.678

$$= 0.678$$

$$c_1 = -1.965$$

$$Av_1 = (-1.965)v_1 + (0.678)v_2$$

$$Av_2 = \begin{bmatrix} 0.57 \\ 0.79 \\ -0.22 \end{bmatrix} = \begin{bmatrix} 0 \\ .75 \\ -.66 \end{bmatrix} c_1 + \begin{bmatrix} 0.84 \\ 0.36 \\ 0.4 \end{bmatrix} (0.679)$$

$$0.57 = c_2(0.84)$$

$$c_1 = 0.727$$

$$c_2 = 0.679$$

$$Av_2 = \begin{bmatrix} 0.57 \\ 0.79 \\ -0.22 \end{bmatrix} = v_1 \cdot 0.727 + v_2 (0.679)$$

$$Av_1 = (-1.965)v_1 + (0.678)v_2$$

$$Av_2 = v_1 \cdot 0.727 + v_2 (0.679)$$

$$(v_1, v_2) (A \text{ restricted to } W)_{(v_1, v_2)}$$

$$= \begin{bmatrix} -1.965 & 0.679 \\ 0.678 & 0.61 \end{bmatrix} \begin{matrix} \curvearrowright \\ \downarrow \end{matrix} ??$$

$$\begin{matrix} \text{evec} \nearrow [-0.97, 0.24] \\ \nwarrow \text{eval } -2.13 \end{matrix}$$

$$-0.97 v_1 + 0.24 v_2$$

$$\begin{matrix} \perp \swarrow [-0.24, -0.97] \\ \searrow \text{eval } 0.778 \end{matrix}$$