

Math 3120, Linear Algebra, Spring 2026, Exam 1 practice sheet

1. Consider the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$.
 - (a) Exhibit a sequence of row operations which puts A into reduced row echelon form.
 - (b) Show how to write A as a product of elementary matrices (matrices which correspond to elementary row operations).
2. Consider the following row operations on a system of 2 equations:
 - exchange the two equations,
 - multiply the first equation by $1/3$,
 - subtract the first equation from the second,
 - multiply the second equation by 2,
 - add the second equation to the first.
 - (a) Suppose that A is a 2×2 matrix which, after applying the above row operations in order, yields the identity matrix $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What is A^{-1} ?
 - (b) If A and B are both 2×2 matrices such that the application of these row operations puts them both into reduced row echelon form, does it follow that $A = B$? Why or why not?
3. Consider the vectors $v_1 = (1, 0, 1)$, $v_2 = (1, 2, 2)$ in $\mathbb{R}^3$. Describe all vectors which are perpendicular to both v_1 and v_2 .
4. Let P be the plane in $\mathbb{R}^3$ consisting of those vectors which are perpendicular to the vector $(2, 1, 3)$.
 - (a) Find the vector in P which is closest to $v = (-1, 0, 1)$.
 - (b) Calculate the distance between P and v .

5. (a) Consider the vector $v = (2, 1, 3)$. What matrix represents the linear transformation $D : \mathbb{R}^3 \rightarrow \mathbb{R}^1$ sending a vector w to the dot product $v \cdot w$?

- (b) Consider the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by the matrix $\begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$. If we denote the columns of T by w_1, w_2 show that

- Every vector u in the image of T can be written in the form $aw_1 + bw_2$.
- The vectors w_1 and w_2 are independent.

- (c) Give a matrix representation for the composition $D \circ T$.

- (d) The kernel of $D \circ T$ can be represented as the collection of vectors which are perpendicular to a vector w . What is such a vector w ?

6. Consider the following sequence of linear transformations in $\mathbb{R}^2$

- swap the x and y axes
- rotate 90° counterclockwise
- $(x, y) \mapsto (x, y - 2x)$
- rotate 180°
- swap the x and y axes

Find a matrix representing the linear transformation obtained by doing this full sequence of operations in order.

7. Suppose we have an $m \times n$ matrix A whose columns correspond to vectors $v_1, \dots, v_n$ and whose rows correspond to vectors $w_1, \dots, w_m$. If all of the vectors v_i are parallel to each other, is the same necessarily true about the w_i 's?

Either explain why this must always be true, or provide a counterexample.