

MATH 3120, Linear Algebra, Spring 2026, Exam 2

Review Sheet (corrected!)

1. Let A be an $n \times n$ matrix, and suppose $x_1, \dots, x_m$ is a basis for the space of solutions to $Ax = 0$. For a vector $v \in \mathbb{R}^n$ how would you find a vector v' which is a solution to $Ax = 0$, and as close as possible to v ?
2. Let A be a matrix whose columns are the vectors $w_1, \dots, w_n$ and whose rows are the vectors $v_1, \dots, v_m$. Let W be the span of the vectors $w_1, \dots, w_n$ and let U be the span of the vectors $v_1, \dots, v_m$. Let T is the linear transformation corresponding to A .

(a) Show that the image of T is W

(b) Show that the kernel of T is $U^\perp$, the set of vectors perpendicular to each vector in U .

3. Suppose you are given vectors $v_1, \dots, v_m \in \mathbb{R}^n$ and a vector $a \in \mathbb{R}^n$.

Describe how you would find a matrix A and a vector b such that the solutions to $Ax = b$ exactly consists of those vectors $v \in \mathbb{R}^n$ such that the vector $v + a$ is perpendicular to each v_i .

4. Consider the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 0 & 1 & 2 \\ 0 & 4 & 2 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Decide whether or not this matrix is invertible (hint: use the determinant!)

5. (this was rewritten!) Let A be an $m \times n$ matrix, and suppose $x \in \mathbb{R}^n$ is a vector.

(a) (stupid trick question) If the equation $Ax = b$ is inconsistent, can we always change b to some other vector b' so that $Ax = b'$ is consistent?

(b) If so, how can we find the vector b' which is as close as possible to b so that $Ax = b'$ has a solution?

6. Consider the vectors $v_1 = (3, 1, 0, 0)$, $v_2 = (1, 2, 1, 1)$, $v_3 = (2, 2, 0, 1)$. Let $V \subset \mathbb{R}^4$ be the span of these vectors. Check to see whether or not $(1, 0, 0, 0)$ is in V .

hint: use the determinant and the various computational shortcuts we have seen!

7. (this was rewritten!) Suppose T is a linear transformation from $\mathbb{R}^p$ to $\mathbb{R}^n$ given by a matrix A , and S is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^q$ given by a matrix B . Let $a \in \mathbb{R}^n$. How might you solve for a vector y of the form $Ax + a$ so that By is as small as possible?
8. (this was rewritten!) Suppose T is a linear transformation from $\mathbb{R}^p$ to $\mathbb{R}^n$ given by a matrix A , and S is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^q$ given by a matrix B . Let $b \in \mathbb{R}^q$ be a vector. How might you solve for a vector y of the form Ax so that By is as close as possible to b ?
9. Consider the vectors $(1, 2, 0)$ and $(1, 1, 1)$ in $\mathbb{R}^3$ and let $V \subseteq \mathbb{R}^3$ denote their span.
- (a) find an orthonormal basis for V
 - (b) find a 3×3 matrix representing the projection onto V .
10. Suppose W and V are vector spaces, and $T : V \rightarrow W$ is a linear transformation. Suppose that T is injective.
- (a) Show that if W has an inner product $\langle \cdot, \cdot \rangle$ then we may define an inner product on V by defining, for $v_1, v_2 \in V$, $\langle v_1, v_2 \rangle = \langle T(v_1), T(v_2) \rangle$.
 - (b) Show that if T is not injective, this will not give an inner product.