

Math 3120, Linear Algebra, Spring 2026, Worksheet 7

April 15, 2026

Name _____

For this worksheet, you will want access to a computer of some sort to be able to iterate powers of matrices!

1. Consider the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

- (a) Find the largest eigenvalue of the matrix A by choosing a vector v and iterating the function $v \mapsto Av/\|Av\|$.
- (b) Let v_0 be the eigenvector you found in the previous part. What is its eigenvalue?
- (c) Write down a vector v_1 which is perpendicular to v_0 . Is v_1 an eigenvector?
- (d) What will happen if we iterate the function $v \mapsto Av/\|Av\|$ starting with v_1 ? Check to see if you are correct.

2. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{bmatrix}$

- (a) Find a basis for $\mathbb{R}^3$ consisting of eigenvectors for A .
- (b) Find an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix.

3. Consider the matrix $A = \begin{bmatrix} 2 & -1 & -2 \\ -1 & 1 & 3 \\ -2 & 3 & 1 \end{bmatrix}$

(a) Find a unit length vector v_0 which is an eigenvector for A with largest possible eigenvalue.

(b) Find a basis for the subspace W of vectors $w \in \mathbb{R}^3$ such that $v_0 \cdot w = 0$.

(c) Find an orthonormal basis v_1, v_2 for W .

(d) Convince yourself that if $w \in W$ then $Aw \in W$, and so A defines a linear transformation from W to itself. Find a matrix representation B for A 's action on W in terms of the basis v_1, v_2 .

(e) Find an orthonormal basis of eigenvectors w_1, w_2 for B .

(f) Find an orthonormal basis of eigenvectors for A (these are the vectors in $\mathbb{R}^3$ corresponding to your eigenvectors from the previous part, which were written in terms of v_1, v_2 . Now you need to write them in terms of the standard basis).