# Examples of hypothesis testing

$H_0$ coin is fair

$H_1$ $P(\text{heads}) > 1/2$

test: flip twice.

HH $\longrightarrow$ $H_1$ biased

else $\longrightarrow$ $H_0$ fair.

$P(\text{type 1 error}) = (H_0 \text{ true, but we reject it})$

$1/4 \quad \alpha$

$P(\text{type 2 error}) = (H_1 \text{ true but we pick } H_0)$

if $P(\text{heads}) \sim \frac{1}{2} + \varepsilon$

$P(\text{type 2}) \sim \frac{3}{4}$

$P(\text{type 2 error}) \leq \frac{3}{4} \quad \beta \quad 1-\beta$

$H_0$ fair

$H_1$ $P(\text{heads}) \geq 90\%$.

flip twice.

HH $\longrightarrow H_1$

else $\longrightarrow H_0$

$P(\text{type 1}) = 1/4$

$P(\text{type 2}) = P(TT, TH, HT \mid \theta \geq 90\%)$

$\leq (.1)^2 + (.1)(.9) + (.9)(.1)$

$= .19 = 19\%$.

want $P(\text{type 1}) \leq 1\%$

flip 7 times: $P(H^7) = \frac{1}{128}$

$H_0$ - fair $H^7 \rightarrow H_1$ $P(\text{type 1}) = \frac{1}{128}$

$H_1$ $P(H) \geq 90\%$ else $\rightarrow H_0$ $P(\text{type 2}) \leq 1 - (.9)^7$

$\approx 1 - .48$

$\approx 52\%$

significance level $\alpha = \frac{1}{128}$

power $1-\beta = .48$

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Instead?

$H_1$ if no more than 1 tail

$H_0$ else.

$P(\text{type 1}) = \left(\frac{1}{2}\right)^7 + 7\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^7$

$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\approx 6\%$

$\alpha = .06$

$P(\text{type 2}) \leq 1 - (.9)^7 - 8(.9)^6(.1)$

$\approx 1 - .9 \approx 10\%$

$\beta = .9$

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