$X \to$ uniform on $[a,b]$

$$f(x) = \begin{cases} c & x \in [a,b] \\ 0 & \text{else} \end{cases}$$

$$\int f(x)\,dx = \int_a^b c\,dx = 1$$

$$= \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{else} \end{cases}$$

Gamma Random variable

exp: p.d.f. $f(x) = \lambda e^{-\lambda x}$

$$f(x) = C x^a e^{-bx}$$

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

$\alpha = 1 \leftrightarrow$ exponential ($\beta = \lambda^{-1}$)

Gamma function

ex: $\alpha = 1$ $\beta = \lambda^{-1}$ exponential

$2\alpha = \nu$ $\beta = 2$ "Chi-square distribution with ν degrees of freedom"

integer ≥ 1

$$M_X(t) = (1-\beta t)^{-\alpha}$$

Given independent, exponential vars $X_1, \dots, X_n$, $f(x) = \lambda e^{-\lambda x}$

$$M_{X_1 + \dots + X_n}(t) = \prod M_{X_i}(t)$$

$$= \prod_{i=1}^{n} (1 - \lambda^{-1} t)^{-1}$$

$$= (1 - \lambda^{-1} t)^{-n}$$

$\sum_{i=1}^{n} X_i$ is a gamma varble w/ $\alpha = n$, $\beta = \lambda^{-1}$

Normal random variables

X a normal random var $\longleftrightarrow$ p.d.f.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

variance σ^2

mean μ

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

X_1 normal σ_1^2, μ_1

X_2 --- σ_2^2, μ_2

$$M_{X_1+X_2}(t) = M_{X_1}(t) M_{X_2}(t)$$

$$= e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2} e^{\mu_2 t + \frac{1}{2}\sigma_2^2 t^2}$$

$$= e^{(\mu_1+\mu_2)t + \frac{1}{2}(\sigma_1^2+\sigma_2^2)t^2}$$

$X_1 + X_2$ is normal w/ mean $\mu_1 + \mu_2$

and variance $\sigma_1^2 + \sigma_2^2$

$X_1, X_2, X_3, \ldots, X_n, \ldots$ identical indep $\leftarrow$ mean μ
variance σ^2

$$\mu\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \mu(X_i)$$

$$E\left[\sum X_i\right] = \sum E[X_i]$$

$$\text{''}$$

$$\mu\left(\sum X_i\right)$$

$$Var\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n Var\, X_i$$

$$\mu(\lambda X) = \lambda\, \mu(X)$$

$$Var(\lambda X) = \lambda^2 \mu(X)$$

$$X = \sum_{i=1}^n X_i \qquad \mu(X) = n\mu \qquad \sigma_X^2 = Var\, X = n\sigma^2$$

$$\frac{X - n\mu}{\sqrt{n}\,\sigma}$$ $\leftarrow$ random var w/ mean 0 & variance 1.

Central limit: c.d.f. of $\dfrac{\left(\sum_{i=1}^n X_i\right) - n\mu}{\sqrt{n}\,\sigma}$

approaches cdf of Z as $n \to \infty$

Example:

Catch fish in river   mean 5.3 lbs

std dev 1.2   Variance ~ 1.4

After catch 10 fish, what's the prob. that we have $\geq$ 50 lbs of fish?

$$X = X_1 + \cdots + X_{10}$$

$$\boxed{X \geq 50}$$

$$X - 53 \geq 50 - 53$$

$$\frac{X - 10\mu}{\sqrt{n}\,\sigma} \approx Z$$

$$Z \approx \frac{X-53}{\sqrt{10}\,(1.2)} \geq \frac{-3}{\sqrt{10}\,(1.2)} \approx -.8$$

$$P(Z \geq -.8) \approx \boxed{79\%}$$