General clarifications

Simple: population has specific distribution —

e.g. • Bernoulli population $\theta = 3/4$

• Normal $\mu = 100$ $\sigma^2 = 40$

Composite: dist. not completely specified

e.g. Bernoulli pop w/ $\theta \geq 3/4$

normal, $\mu = 100$, σ^2 = unknown

---- $\mu \in [80, 120]$, $\sigma^2 = 40$

We can't generally compute exact probabilities given a composite hypothesis.

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normal

H_0: $10 \leq \mu \leq 15$ $\sigma^2 = 25$ $n = 5$

H_1: $\mu = 40$ $\sigma^2 = 25$

$P(\bar{X} > 12 \mid H_0)$ = not calculatable

$P(\bar{X} > 12 \mid H_1)$ = calculatable.

$P(\bar{X} > 12 \mid \mu)$ as a function of μ ($\sigma^2 = 25$, $n = 5$ fixed) can consider min & max values

this fcn takes on [10, 15]

could find statement:

$$? \le P(\bar{X} > 12 \mid H_0) \le ?$$

Example

H_0: fair coin

H_1: $P(\text{heads}) > \frac{1}{2}$ $\quad \theta \in (\frac{1}{2}, 1]$

experiment: flip 10 times, if all heads conclude H_1

if not all heads, conclude H_0

$$P(\text{type } 1) = P(\text{heads}^{10} \mid H_0) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

$$P(\text{type } 2) = P(\text{heads}^{<10} \mid H_1)$$

$$= 1 - P(\text{heads}^{10} \mid H_1) \in \left[0, \frac{1023}{1024}\right)$$

$$P(\text{heads}^{10} \mid \theta) = \theta^{10}$$

$$P(\text{heads}^{<10} \mid \theta) = 1 - \theta^{10} \quad \text{if } \theta \in (\tfrac{1}{2}, 1]$$

$\theta = 1$ min value of $1 - 1 = 0$

$\theta \to \frac{1}{2}$ approaches supremum $1 - \frac{1}{1024}$

$$= \frac{1023}{1024}$$

if $\theta = 50.0001\%$ (H_1)