Math 477, Practice sheet for Exam 2

This exam will cover sections 5.1–5.5, 5.6.1, 6.1–6.5, 7.1–7.2 and 7.5.

It is possible that we will need to rely on various facts and ideas from the earlier sections as well, however I will not aim to directly test these in isolation.

The problems below do not necessarily cover all the topics completely, but will hopefully still be helpful as a reminder of some of the material covered. Please also look through prior worksheets and homework assignments.

1. Suppose n numbers $X_1, X_2, \ldots, X_n$ are chosen from a uniform distribution on $[0, 10]$. We say that there is an increase at i if $X_i < X_{i+1}$. Let I be the number of increases. Find $E[I]$.

2. Suppose that the time until a hurricane in months in a particular region in a given year is represented by an exponential random variable X with density function $f(x) = 12e^{-12x}$. Suppose the time to rebuild after a hurricane is given by a random variable Y uniformly distributed on the interval $[3, 8]$. Find the expected time elapsed until a hurricane occurs and rebuilding is complete.

3. Suppose that the time between hurricanes in months in a particular region in a given year is represented by an exponential random variable X with density function $f(x) = 12e^{-12x}$.

4. Suppose that X and Y are uniformly distributed independent random variables in $[0, 1]$. Find $E[X^2+Y^2]$.

5. Suppose numbers A, B, C are picked independently and uniformly from the interval $[0, 1]$. What is the probability that the equation $Ax^2 + Bx + C = 0$ has two real roots?

6. Suppose that a game is played with a group of $2n$ people twice. Each time, the players are randomly paired. Let X be the number of pairs which occur in both the first and second game. Find $E[X]$.

7. If you roll a fair die, what is the expected number of rolls necessary in order to get a '1'?

8. If you roll a fair die, what is the expected number of rolls necessary in order to get both a '1' and a '2' (not necessarily in that order)?