Bestiary of random variables

Binomial Random Variables w/ parameters (p,n)

$$P(X=i) = \binom{n}{i} p^i (1-p)^{n-i} \quad i = 0,1,\dots,n$$

Interpretation: p prob of success $\quad$ $X = \#$ successes.
n # trials

$$E[X] = pn \qquad Var(X) = np(1-p)$$
(will derive later)

Poisson Variable

X Poisson w/ parameter λ means

$$P(X=i) = e^{-\lambda}\frac{\lambda^i}{i!}.$$

Interpretation: Suppose have radioactive material emitting on average λ α-particles every second

model w/ binomial

ex: $\lambda = 2$

every $1/10$'th of a second expect $\sim \frac{1}{5}$ of a particle prob $\frac{1}{5}$ of emission each $1/10$ sec.

$X = \#$ in one second $\approx$ binomial $p = \frac{1}{5}$

$n = 10$

as # subdivisions $\to \infty$

get Poisson w $\lambda = np$

$E[X] = \lambda$

$\mathrm{Var}(X) = \lambda$

$np(1-p) \leftrightarrow \lambda(1 - \frac{\lambda}{n})$

$np \leftrightarrow \lambda \quad p \leftrightarrow \frac{\lambda}{n}$

Geometric Random Variable

$$P(X = i) = (1-p)^{i-1} p \quad X = 1, 2, \ldots$$

w/ parameter p.

Interpretation: p = prob. of success

X = # of trials until success.

$E[X] = \frac{1}{p} \quad \mathrm{Var}(X) = \frac{1-p}{p^2}$

Negative Binomial Variable

X is a neg. binomial variable w/ params (r,p)

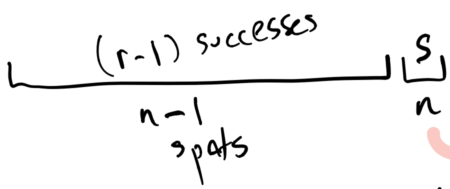
if $P(X=n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$

Interpretation:

prop p of success

X = # trials until r successes.

($r=1$, get geometric variable)



if need n times to get r successes

$$P(X=n) = \binom{n-1}{r-1} p^{r-1} (1-p)^{(n-1)-(r-1)} \cdot p$$

$$= \binom{n-1}{r-1} p^r (1-p)^{n-r}$$

$$E[X] = \frac{r}{p} \qquad Var(X) = \frac{r(1-p)}{p^2}$$