Recall: Normal random variable X w/ parameters (μ, σ^2) has prob. density function

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

Observe $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$u = x - \mu$$
$$du = dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-u^2/2\sigma^2}\,du$$

$$\frac{u}{\sigma} = v \qquad \frac{du}{\sigma} = dv$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-v^2/2}\,dv$$

$$\left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)$$

$$= \left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}\,dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2/2} e^{-y^2/2}\,dx\,dy$$

$$\left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)^2 = 2\pi$$

$$= \iint e^{-(x^2+y^2)/2}\, dx\, dy$$

$$(r^2 = x^2 + y^2)$$

$$x = r\cos\theta \qquad dx\,dy = r\,dr\,d\theta$$

$$y = r\sin\theta$$

$$= \int_{\theta=0}^{2\pi} \underbrace{\int_{r=0}^{\infty} e^{-r^2/2}\, r\, dr}\, d\theta \;=\; \int_{\theta} \Big(\underbrace{\int_{u=0}^{\infty} e^{-u}\, du}_{1} \Big) d\theta$$

$$u = r^2/2$$

$$du = r\,dr$$

$$\underbrace{}_{2\pi}$$

$$\left[-e^{-u}\right]_0^{\infty}$$

$$= -(0-1) = 1$$

$$\int_0^{2\pi} 1\, d\theta = 2\pi - 0 = 2\pi$$

<u>Recall</u> if X is binomial, (n,p)

$n \gg 0$, $\lambda = np$ is "relately small" compared to n.

then $X \approx$ Poisson w/ parameter λ.

X is binomial, (n,p) λ is comparable to n.

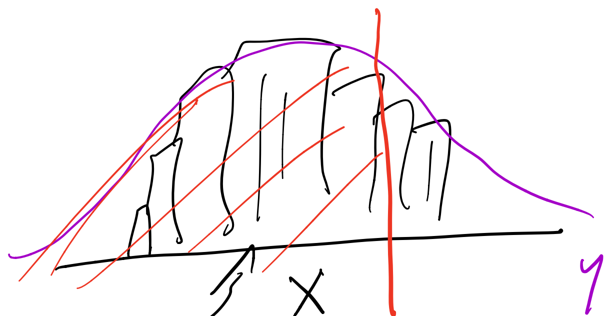
then $P(X < a) \approx P(Y < a)$

"a regular size"

Y is a normal random variable w/

$\mu = np = \lambda$

$\sigma^2 = np(1-p)$



"continuity correction"

Common approach for approximating

$$\boxed{P(X=a) \approx P\left(a-\tfrac{1}{2} \le Y \le a+\tfrac{1}{2}\right)}$$

rectangle of width one around a in

<u>Recall</u> $E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$

and as with discrete case (in analogy)

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

- If 12% of population is left handed, given 200 people, what's the prob that (at least) 20 are left handed?

X = # of left handed people in random gp of 200

binomial (n,p) $\quad n = 200$

$p = 12/100 = 6/50 = 3/25.$

$\lambda = np = 24 = \mu$

$\sigma^2 = np(1-p) = 24\,(22/25)$

$Y = \text{Normal}\ (\mu, \sigma^2) \qquad \approx 21$

$$P(X=20) \approx P\left(20-\tfrac{1}{2} \le Y \le 20+\tfrac{1}{2}\right)$$

$$= \int_{20-\frac{1}{2}}^{20+\frac{1}{2}} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-24)^2/2(21)}\, dx$$

σ: 4.7

approx w/ $\boxed{1}$ rectangle : $1 \cdot \dfrac{1}{\sqrt{2\pi}\sqrt{21}} e^{-(20-24)^2/2(21)}$

$$= \frac{1}{\sqrt{42\pi}} e^{-4^2/42}$$

$$\approx \frac{1}{11} e^{-16/42}$$