Suppose we choose 2 points in $[0,2]$ independently, each uniformly distributed.

what's the prob that the points are no more than 1 unit apart?

$\boxed{\frac{3}{4}}$ (Victor)

X first point's position

Y second

$$P(X \le a) = \begin{cases} 0 & a < 0 \\ a/2 & a \in [0,2] \\ 1 & a > 2 \end{cases}$$

$$\int_0^a \frac{1}{2}\,dx$$

Suppose we roll 2 dice $X \in \{1,\dots,6\}$

$Y \in$

Prob results are at most 1 apart?

$$\frac{1}{36} = P(X=a, Y=b) \qquad a,b \in \{1,\dots,6\}$$

$$P(\text{within } 1) = \sum_{\substack{(a,b) \\ |a-b| \le 1}} \underbrace{P(X=a, Y=b)}_{= 1/36}$$

<u>First Step</u> Suppose we have 2 random vars X, Y

$$P(X \le a; Y \le b) = P(EF)$$

$$E = \{X \le a\} \qquad F = [Y \le b]$$

<u>Def</u> $F(a,b) \equiv P(X \le a; Y \le b)$ called the cumulative dist. function for X, Y.

<u>Note</u>: from this, can get dist. fcns for X & Y individually

$$F_X(a) = P(X \le a) = P(X \le a, \text{ no cond on } Y)$$

$$= \lim_{b \to \infty} P(X \le a; Y \le b)$$

$$= \lim_{b \to \infty} F(a,b) \equiv \text{"}F(a,\infty)\text{"}$$

If X, Y discrete, then $F(a,b) = \sum_{\substack{x \le a \\ y \le b}} P(X = x; Y = y)$ // notation.

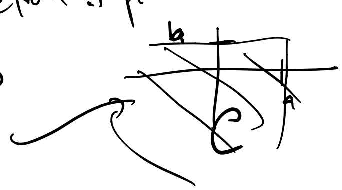
$$= \sum_{\substack{x \le a \\ y \le b}} p(x,y)$$

in general, X, Y discrete

$$P((X,Y) \in C) = \sum_{(x,y) \in C} p(x,y)$$

↑ collection of pts in $\mathbb{R}^2$

$X \le a; Y \le b$



Def If X & Y are random vars, we say they are jointly continuously distributed if

$$P\left((X,Y)\in C\right) = \iint_{(x,y)\in C} f(x,y)\,dx\,dy$$

joint prob. density fcn

same $f(x,y)$

equiv. defs.

for example:

cum. dist fcn (joint)

$$F(a,b) = P(X\le a;\ Y\le b) = \int_{y=-\infty}^{y=b}\int_{x=-\infty}^{x=a} f(x,y)\,dx\,dy$$

Note: by FTC, $f(x,y) = \dfrac{\partial^2}{\partial x\,\partial y} F(x,y)$

Back to our two points in $[0,2]$

X pt 1

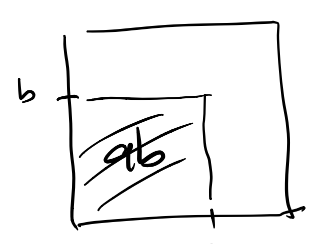
Y pt 2

X, Y unif. dist.

since X & Y independent

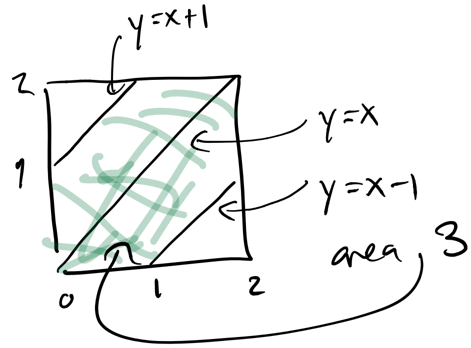
assume $a,b\in[0,2]$

$$\begin{aligned} F(a,b) &= P(X\le a;\ Y\le b)\\ &= P(X\le a)\,P(Y\le b)\\ &= \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) = \frac{ab}{4}\\ &= \int_{y=0}^{b}\int_{x=0}^{a} \frac{1}{4}\,dx\,dy \end{aligned}$$



$\Rightarrow X$ & Y are cont. jointly distributed w/ density $\frac{1}{4}$ in $[0,2]\times[0,2]$

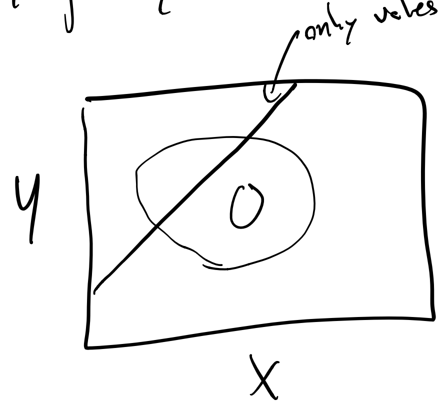
$$\iint_{\substack{|x-y|<1 \\ x,y\in[0,2]}} \frac{1}{4}\,dx\,dy = \frac{3}{4}$$



Given a cont rand var X

$Y = X + 1$.

are X & Y jointly cont? <u>no</u>



$$\int_a^a f(x)\,dx = 0$$

Suppose X, Y random vars, jointly cont. distributed.

consider $Z = X + Y$. what is p_Z?

$$F_Z(a) = P(Z \le a) = P(X+Y \le a)$$
$$= P(Y \le a - X)$$
$$= \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{a-x} f_{X,Y}(x,y)\,dy\,dx$$

$$p_Z(a) = \frac{d}{da} \int_{-\infty}^{\infty} \int_{y=-\infty}^{a-x} f_{X,Y}(x,y)\,dy\,dx$$

$$= \int_{x=-\infty}^{\infty} f_{X,Y}(x, a-x)\,dx$$

<u>Df</u> X & Y are called <u>independent</u> if

$$f_{X,Y}(a,b) = f_X(a) f_Y(b)$$

in this case $Z = X+Y$, above gives $f_{X+Y}(a) = \int_{x=-\infty}^{\infty} f_X(x) f_Y(a-x)\,dx$

$$= f_X * f_Y(a)$$