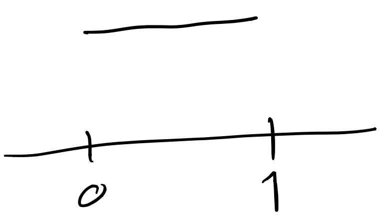


X, Y

$$f(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{else} \end{cases}$$

$X+Y$

recall: if X has pdf $f(x)$ $\Rightarrow X+Y$ has $h(x)$

Y " " $g(x)$ (assuming independent)

where $h(x) = \int_{t=-\infty}^{\infty} f(t)\,g(x-t)\,dt$

"convolution"

integrate our values where arguments add to x.

$h(x) = \int_{t=-\infty}^{\infty} f(t)\,f(x-t)\,dt$

$x+y$ x y

if $\boxed{0 \le x \le 1}$

$f(t)$ t $-\infty$

$0 \le t \le 1$ $f(t)=1$

$\int_0^1 1 \cdot f(x-t)\,dt$

$0 \le t \le x \rightsquigarrow f(x-t) = 1$

$t > x \rightsquigarrow f(x-t) = 0$

$= \int_0^x 1\,dt = x = h(x)$

if $1 \le x \le 2$

$f(t)$

$0 \le t \le 1$

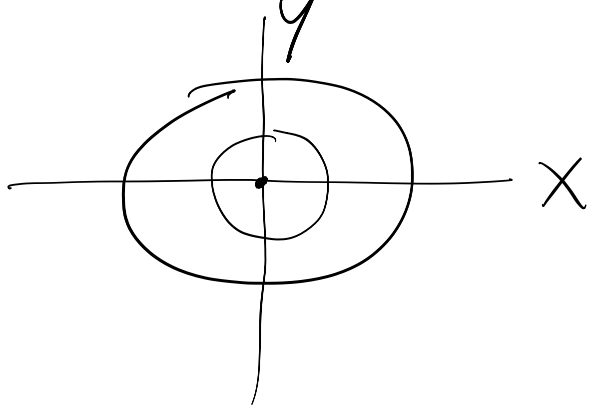
$\int_0^1 1 \cdot f(x-t)\,dt$

$\boxed{\begin{array}{c} x-1 \le t \le 1 \\ f(x-t) = 1 \end{array}}$

$\int_{x-1}^1 1\,dt = 1-(x-1)$
$= 2-x$

$$h(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \end{cases}$$

Drop a ball at target



Assume - independent, and that joint p.d.f. $f(X,Y)$ depends only on R = dist. from 0.

turns out $\Rightarrow$ X & Y normally distributed.

$$f(x,y) = f_X(x) f_Y(y) = g(x^2+y^2) \qquad \text{EQ 1}$$

(independence)

$$\frac{\partial}{\partial x} \Rightarrow f_X'(x) f_Y(y) = g'(x^2+y^2)\,2x \qquad \text{EQ 2}$$

$$\frac{EQ_2}{EQ_1} \qquad \frac{f_X'(x)\cancel{f_Y(y)}}{f_X(x)\cancel{f_Y(y)}} = \frac{g'(x^2+y^2)\,2x}{g(x^2+y^2)}$$

Claim LHS constant.
$$\frac{f_X'(x)}{f_X(x)\,2x} = (x^2+y^2) \text{ stuff.}$$

$x_1 \quad x_2$

$x_1^2 \quad x_2^2$ change y_1, y_2

so that $x_1^2+y_1^2 = x_2^2+y_2^2$

then RHS <u>same</u>

so LHS same for x_1 & x_2 any values.

$$\frac{f_X'(x)}{f_X(x)\,2x} = C \quad \text{const.}$$

$$\frac{f_X'(x)}{f_X(x)} = 2xC = Cx$$

$$'' \quad \frac{d}{dx}\log(f_X(x))$$

$$\log(f_X(x)) = \frac{1}{2}Cx^2 + D$$

$$f_X(x) = e^D e^{\frac{1}{2}Cx^2} \Rightarrow \underline{\text{normal}}$$

$X \quad Y \qquad \cancel{X+Y}$

$$\sqrt{X^2+Y^2}$$

Frst step X^2+Y^2

recall: exp. dist. — "wait tme in Poisson process"

wait tme until n occurrences exp. process

Γ-dist. w/ parameters λ, n