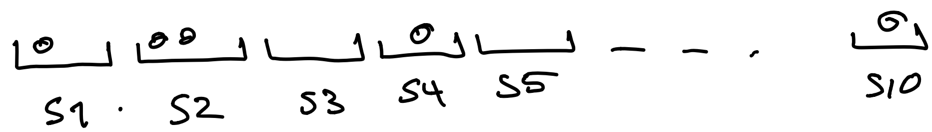
5 Identical balls distributed to 10 students



Insert dividers to keep student's property to themselves

$\overset{o}{S1} \mid \overset{oo}{S2} \mid \; - \; \ldots \; \mid \overset{o}{S10}$

pattern of balls & dividers

0|00||0||||||0   9 dividers

5 balls

# of such patterns

= # ways to choose which of the 14 symbols are balls & which are dividers

$$= \binom{14}{5,9}$$

Abstract Problem:

How many ways can we choose nonnegative numbers $x_1, x_2, \ldots, x_n$ such that $\sum x_i = r$

some r

answer = need $n-1$ separators & r balls

$$\binom{r+(n-1)}{r,\ n-1} = \binom{r+n-1}{r}$$

Multinomial Coefficients

Counts the number of ways to distribute objects into groups.

Given n labelled/distinguishable objects to distribute into k labelled/distinguishable groups of size $n_1, n_2, \ldots, n_k$ $\quad (\sum n_i = n)$

\# of ways is called $\binom{n}{n_1, n_2, \ldots, n_k}$

<u>Ex</u> $\binom{n}{1,1,1,\ldots,1} = n!$

$$\binom{n}{k,\ n-k} = \binom{n}{k} = \binom{n}{n-k} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{10}{3,3,2,2} = \binom{10}{3}\binom{7}{3,2,2} = \binom{10}{3}\binom{7}{3}\binom{4}{2,2}$$

first choose 3 fr gp 1 $= \binom{10}{3}\binom{7}{3}\binom{4}{2}\binom{2}{2}$

then distribute remaing 7

$$= \frac{10!}{3!\cancel{7!}} \frac{\cancel{7!}}{3!\cancel{4!}} \frac{\cancel{4!}}{2!\cancel{2!}} \frac{\cancel{2!}}{2!0!}$$

$$= \frac{10!}{3!3!2!2!}$$

General Formula $\binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1! n_2! \ldots n_k!}$

why are these called multinomial coefficients?

$(x_1 + x_2 + \cdots + x_k)^n$

coeff of $x_1^{n_1} x_2^{n_2} \cdots x_k^{n_k}$ is $\binom{n}{n_1, \ldots, n_k}$

$(x_1 + x_2 + x_3 + x_4)^{10}$ coeff of $x_1^3 x_2^3 x_3^2 x_4^2$

foil!

$(x_1+x_2+x_3+x_4)(x_1+x_2+x_3+x_4)(x_1+x_2+x_3+x_4)\ldots\ldots$

3 3

$$x_1^3 x_2^3 x_3^2 x_4^2$$

choose which 3 terms cont. x_1 $\binom{10}{3,3,2,2}$

" 3 - - - x_2

2 " — - x_3

2 · x_4

$\boxed{\text{read ch 1}}$!

Probability (Chap 2)

The basic object in probability theory is the probability space

Set S = "sample space"
interpreted as set of possible outcomes of experiments

subsets of S are called "events"

ex: experiment = flip a coin twice.

$S = \{(H,H),(H,T),(T,H),(T,T)\}$

$E = \{(H,H),(T,T)\}$

Formally: A probability space consists of 3 datum

Set S = sample space

(Collection Ω of subsets of S called events)

Function $P : \Omega \longrightarrow \mathbb{R}$

such that some axioms hold:

(0) Ω satisfies some axioms that (won't mention)

often Ω = all subsets

1) $P(E) \in [0,1]$

2) $P(S) = 1$

3) If $E_1, E_2, \ldots, \in \Omega$ and $E_i \cap E_j = \emptyset$

then $P(\bigcup_i E_i) = \sum_i P(E_i)$

Set Theory notation & operations

Fixed set S, $E, F, G \subset S$

$$E^c = S \setminus E = \{x \in S \mid x \notin E\}$$

$$E \cup F = E + F = \{x \in S \mid x \in E \text{ or } x \in F\}$$

$$E \cap F = EF = \{x \in S \mid x \in E \text{ and } x \in F\}$$

Commutativity, associativity, distributivity:

$$E + F = F + E \qquad EF = FE$$

$$E + (F + G) = (E + F) + G \qquad (EF)G = E(FG)$$

$$E(F + G) = EF + EG$$

de Morgan's Laws $\left(E_1 + E_2 + \cdots + E_n\right)^c = E_1^c E_2^c \cdots E_n^c$

$$\left(E_1 E_2 \cdots E_n\right)^c = E_1^c + \cdots + E_n^c$$

$$\boxed{\left(E_1 + E_2\right)^c = E_1^c E_2^c}$$ ← Prove this

Hint

$$A + B = A + BA^c$$