Conditional probabilities (continuous case) reminder

def of cond. prob: $P(E|F) = \dfrac{P(EF)}{P(F)}$

X, Y jointly cont, w/ density $f(x,y)$

$$P(X \in I) = \int_{y=-\infty}^{\infty} \int_{x \in I} f(x,y)\,dx\,dy$$

P(EF)

$$P(X \in I \mid Y = a) = \frac{\int_{x \in I} f(x,a)\,dx}{\int_{x=-\infty}^{\infty} f(x,a)\,dx}$$

$$P(X \in I \mid X < Y) = \frac{P(X \in I \text{ and } X < Y)}{P(X < Y)}$$

$$\frac{\iint_{x \in I,\, x<y} f(x,y)\,dx\,dy}{\iint_{x<y} f(x,y)\,dx\,dy} = \frac{\int_{x \in I} \int_{x}^{\infty} f(x,y)\,dy\,dx}{\int_{x \in \mathbb{R}} \int_{x}^{\infty} f(x,y)\,dy\,dx}$$

Note: $\int_{x\in\mathbb{R}} \int_{x}^{\infty} f(x,y)\,dy\,dx = \int_{y\in\mathbb{R}} \int_{-\infty}^{y} f(x,y)\,dx\,dy$

" $P(E|F) = \dfrac{P(EF)}{P(F)}$ "

$$P(X\in I \mid X<Y,\ Y=a)$$

$$= \frac{\int_{x\in I\cap(-\infty,a)} f(x,a)\,dx}{\int_{x\in(-\infty,a)} f(x,a)\,dy}$$

How to think about this:

if we fix the condition $X<Y$

$$P(E \mid X<Y)$$

$$P\big(X\in I \mid X<Y \,\big|\, Y=a\big)$$

Review sheet #8

$X = \#$ rolls to get 1 or 2

$Y = \#$ rolls to get any specific number

$E(X+Y) = 3 + 6 = 9.$

Recall: if have prob. p of success, expected time until success (geometric) is $\frac{1}{p}$

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1. Choosing $X_1, X_2, \ldots, X_n$ from $[0,1]$

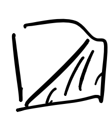
increase at i if $X_i < X_{i+1}$

$I = \#$ increases. $E[I]$

E_i event of an increase at i $\quad I_i$ indicator for E_i

$$I = \sum_{i=1}^{n-1} I_i \qquad P(E_i) = \int_{x_{i+1}=0}^{1}\int_{x_i=0}^{x_{i+1}} 1 \, dx_i \, dx_{i+1} = \frac{1}{2}$$

$$E[I] = \sum_{i=1}^{n-1} E[I_i] = \sum_{i=1}^{n-1} P(E_i) = \frac{n-1}{2}.$$



4. X & Y uniform in $[0,1]$ find $E[X^2+Y^2]$

$$\int_0^1\int_0^1 (x^2+y^2)\,dx\,dy$$

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5. A, B, C uniform on $[0,1]$

$P(Ax^2+Bx+C=0$ has two real roots$)$

$P(B^2-4AC>0)$

$B^2 > 4AC$

$A < \frac{B^2}{4C}$

$$= \iiint_{\substack{A,B,C \\ B^2-4AC>0}} 1\, dA\,dB\,dC$$

$$= \int_{C=0}^{1}\int_{B=0}^{1}\int_{A=0}^{\min\{B^2/4C,\,1\}} dA\,dB\,dC$$

$$= \int_C\int_B B^2/4C\, dB\,dC$$

. . .

addendum: since all vars ≤ 1, given C,

need $B^2 > 4AC \leq 1$ $\quad 1 > B^2 > 4AC$

$0 \leq A \leq \frac{1}{4C}$ $\quad 1 > B > 2\sqrt{AC}$

$$\int_{C=0}^{1}\int_{A=0}^{1/4e}\int_{B=2\sqrt{AC}}^{1} 1 \, \partial B \, \partial A \, \partial C$$