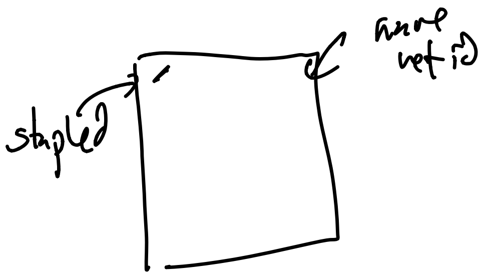
(Roughly Sections 2.4, 2.5)

HW due Friday



Midterm dates

March 1
April 19 } Fridays

OH: Tues 1:25 – 2:25

$\frac{8!}{12!}$ is a great answer.

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Caution:

Please never write $\overset{\text{number}}{1} - E \leftarrow \text{set}$

$EF = E \cap F$ $\qquad E^c$

$F+F = E \cup F$

on the other hand $P(E)$ is a number

so $1 - P(E)$ . .

Reminder of axioms

Prob space: $S$ - sample space

$E \rightsquigarrow P(E) \in [0,1]$

$P(S) = 1$ if $\{E_i\}_{i=1,2,...}$ are disjoint

then $$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

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In particular, if $E \subset S$

$$S = E \cup E^c \qquad E \cap E^c = \emptyset$$

$$\Rightarrow P(E) + P(E^c) = P(S) = 1$$

$$\boxed{P(E^c) = 1 - P(E).}$$

Slight variation:

any $E, F \subset S$

$$\begin{aligned} F &= F \cap S \\ &= F \cap (E \cup E^c) \\ &= (F \cap E) \cup (F \cap E^c) \end{aligned}$$

alt. notation $= FE \cup FE^c$

$F = FE \cup FE^c$ ← disjoint

$$FE \cap FE^c = FEFE^c = F(\underset{\emptyset}{\underset{\shortparallel}{EE^c}}) = \emptyset$$

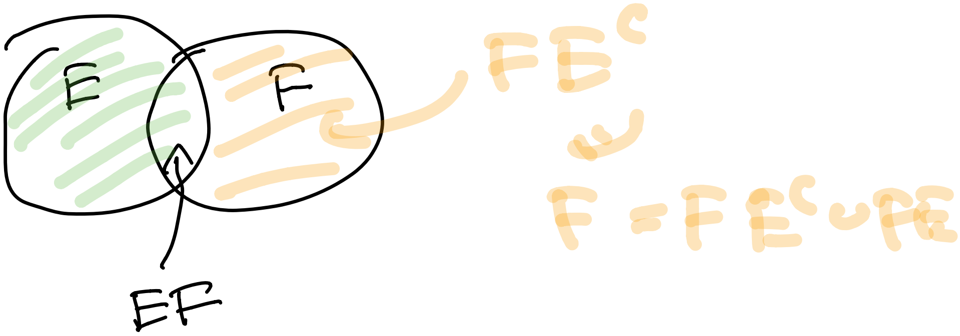
$$\boxed{P(F) = P(FE) + P(FE^c)}$$

$$P(E \cup F) = P(E \cup (FE \cup FE^c))$$

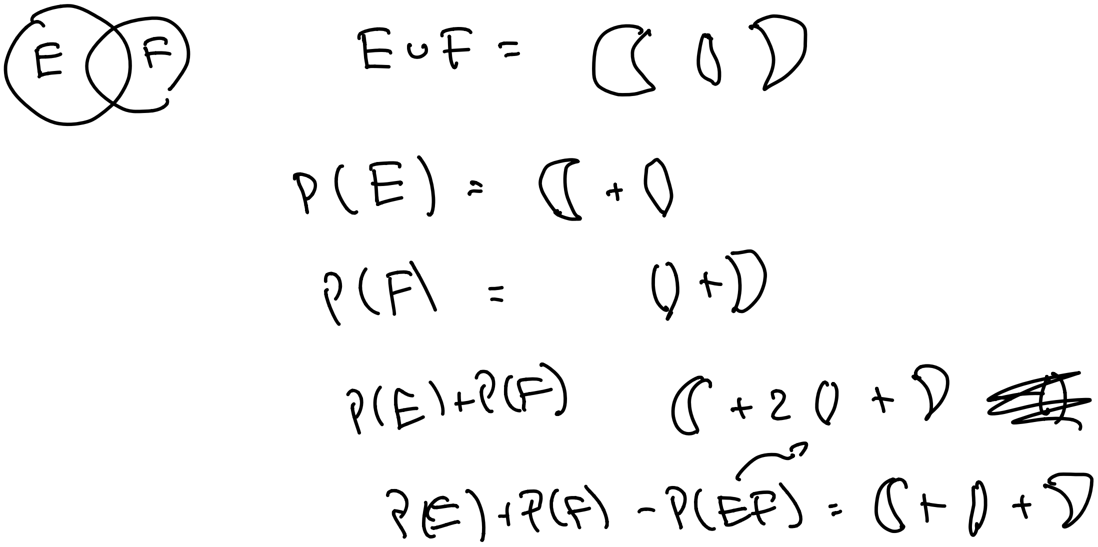
$$= P((E \cup FE) \cup FE^c))$$

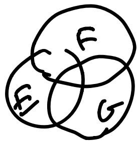
$$= P(E \cup FE^c) = P(E) + \boxed{P(FE^c)}$$

$$\boxed{P(E \cup F) = P(E) + P(F) - P(EF)}$$

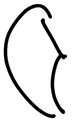


$$F = FE^c \cup FE$$





$P(E) =$ 

$P(E)+P(F)+P(G) =$  + twice 

thrice +3 

EF  + FG + EG

 thrice +3 

$$P(E \cup F \cup G) = P(E)+P(F)+P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$$

thy 1 thy 2

$$P(E \cup F \cup G) = P((E \cup F) \cup G)$$

$$= P(E \cup F) + P(G) - P((E \cup F) \cdot G)$$

$$= P(E \cup F) + P(G) - P(EG \cup FG)$$

$$= \left[P(E)+P(F)-P(EF)\right] + P(G) - \left[P(EG)+P(FG) - P(EGFG)\right]$$

$$= P(E)+P(F)-P(EF)+P(G) - P(EG) - P(FG) + P(EFG)$$

$$\boxed{\begin{aligned} &P(E \cup F \cup G) \\ &\quad = P(E)+P(F)+P(G) - P(EF) - P(EG) - P(FG) \\ &\qquad\qquad\qquad\qquad\qquad + P(EFG) \end{aligned}}$$

$$P(E \cup F \cup G \cup H) = P((E \cup F \cup G) \cup H)$$

$$= P(E \cup F \cup G) + P(H) - P((E \cup F \cup G)H)$$

$$= P(E \cup F \cup G) + P(H) - P(EH \cup FH \cup GH)$$

$$= \left[ P(E)+P(F)+P(G) - P(EF) - P(EG) - P(FG) + P(EFG) \right]$$

$$+P(H) - \left[ P(EH) + P(FH) + P(GH) - -P(EHFH) - P(EHGH) - P(FHGH) + P(EHFHGH) \right]$$

$$= P(E)+P(F)+P(G) - P(EF) - P(EG) - P(FG)$$

$$+P(H) - P(EH) - P(FH) - P(GH) + P(EFH)$$

$$+P(EGH) + P(FGH) - P(EFGH)$$

$= P(E) + P(F) + P(G) + P(H) - P($

"principle of inclusion/exclusion

$$P(\cup E_i) = \sum_i P(E_i) - \sum_{i \neq j} P(E_i E_j)$$

$$+ \sum_{\substack{i \neq j \neq k}} P(E_i E_j E_k) \cdots$$

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## Examples of Probability spaces

$S$ = finite set (say, $N$ elements)

$$P(\{i\}) = \frac{1}{N} \quad i \in S$$

"equally likely outcomes"

$$P(E) = \frac{\#E}{N}$$

example: chance of getting exactly 2 heads after 4 coin tosses.

$$S = \{ (H,H,H,H), (H,H,H,T), \ldots \}$$

$2^4 = 16$ possibilities.

$\# E = \binom{4}{2}$ $\quad$ $E = \{ \text{exactly 2 heads} \}$

$= 6$ $\quad$ $(H,H,T,T)$ - -

$P(E) = \frac{6}{16} = \frac{3}{8}$

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$S_1 =$ outcomes of sequence of 2 dice rolls

$\{(2,6), (1,3), \ldots \}$ $\quad$ $6^2$ possibilities

$S_2 =$ sum of 2 dice rolls

$\{2, 3, \ldots, 12\}$

$P(\text{in } S_2)$ of $3 \longleftrightarrow P(\{(1,2),(2,1)\})$ in $S_1$

$\frac{2}{36}$