Counting example

Select 5 people randomly from 17 people

10 have blue shirts 7 red shirts

what's the probability that we select

3 blue shirts & 2 red ?

"Sample spaces with equally likely outcomes"

Sample space = { subsets of 5 people from the 17 }

$$P(3B, 2R) = \frac{\#\{\text{subsets w/ } 3B, 2R\}}{\#\{\text{subsets w/ 5 people}\}} \quad \begin{matrix} \leftarrow \binom{10}{3}\binom{7}{2} \\ \leftarrow \binom{17}{5} \end{matrix}$$

Alt Sample Space = { sequences of 5 people from 17 }

$$P(3B, 2R) = \frac{\#\{\text{seq w/ } 3B \,\&\, 2R\}}{\#\{\text{seq}\}} \quad \begin{matrix} \leftarrow \binom{10}{3}\binom{7}{2} 5! \\ \leftarrow 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 = \frac{17!}{12!} \end{matrix}$$

$\{\text{seq of 5}\} \longrightarrow \{\text{subset of 5}\}$ $\qquad \#\{\text{seq}\} = 5! \, \#\{\text{subsets}\}$

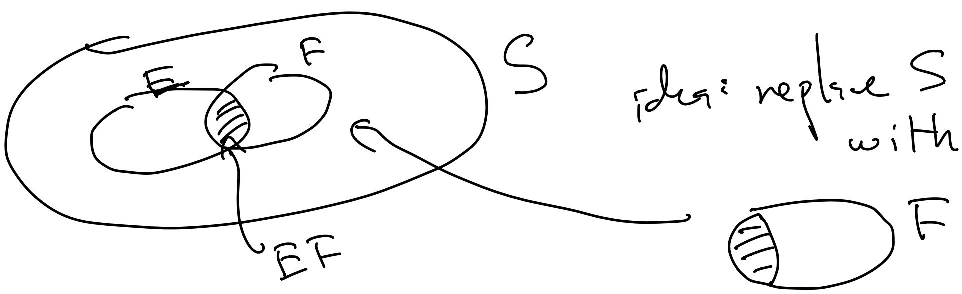
$5! \longrightarrow \cdot$

Conditional Probability

Given events E, F, the conditional probability

$P(E|F)$ is defined as $\dfrac{P(EF)}{P(F)}$ fraction of F which is EF

probability of E given that F has occurred



idea: replace S with F

Ex: Suppose we flip a coin twice, get at least one heads. What's the prob. that both flips are heads?

$$S = \{(H,H),(H,T),(T,H),(T,T)\}$$

$$P(\text{two heads} \mid \geq \text{one head}) = \frac{P(\text{two heads})}{P(\geq \text{one head})} = \frac{(1/4)}{(3/4)}$$

$$= 1/3$$

in case where all outcomes are equally likely:

$$P(E) = \frac{\#E}{\#S}$$

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\#(EF)/\#S}{\#F/\#S}$$

$$= \#EF/\#F$$

$$P(E|F) = \frac{P(EF)}{P(F)}$$

$$\Rightarrow \underline{P(EF) = P(E|F)\,P(F)}$$

50% chance that die w/ roll all 6's

50% chance regular $\quad P(F|E^c) = \frac{1}{6}$

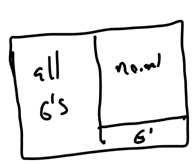
roll 6 first roll

what's the prob that rolls all 6's?

$$P(EF) = P(E) = \frac{1}{2}$$

F roll 6 on first roll $\quad P(E|F)$

E all 6's. $\quad \left(P(F) = \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{6}\right) \quad P(E)$



$$P(F|E) = \frac{P(EF)}{P(E)} = 1$$

$$P(F|E) = \frac{P(EF)}{P(E)} \qquad P(F) = P(FE) + P(FE^c)$$

$$F = F(E \cup E^c) = FE \cup FE^c$$

$E \subset F$

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{(1/2)}{(7/12)} \qquad P(EF) = P(E) = \frac{1}{2}$$

$$P(F) = P(FE) + P(FE^c) = \frac{1}{2} + \frac{1}{12} = \frac{7}{12}$$

$$P(FE^c) = P(F|E^c)P(E^c)$$

$$= \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

$$\boxed{\frac{6}{7}} \; !$$

E – die rolls all 6's $\qquad P(E) = \frac{1}{2}$

E^c – fair die $\qquad P(E^c) = \frac{1}{2}$

F first roll is a 6 $\qquad P(F|E) = 1$

$$P(F|E^c) = \frac{1}{6}$$

P(second roll is 6 | first roll is 6)

$$= \frac{P(\text{both } 6)}{P(\text{first is } 6)} = \frac{P(FG)}{P(F)}$$

E = all 6 die

E^c = fair die

F = first 6

G = second 6

$$P(FG) = P(FGE) + P(FGE^c)$$

$$P(FGE) = P(FG|E)\,P(E)$$