3 people 3 homeworks passed back randomly

$H_1$ person 1 gets thr HW back

$H_2$ "

$H_3$ "

$\frac{2!}{3!}$

$$P(H_1 \cup H_2 \cup H_3) = P(H_1) + P(H_2) + P(H_3)$$

$$- P(H_1 H_2) - P(H_1 H_3) - P(H_2 H_3)$$

$$+ P(H_1 H_2 H_3)$$

$\frac{1!}{3!}$

$\frac{0!}{3!}$

$$3 \cdot \frac{2!}{3!} - 3 \frac{1!}{3!} + \frac{0!}{3!}$$

$$= \binom{3}{1} \cdot \frac{2!}{3!} - \binom{3}{2} \frac{1!}{3!} + \binom{3}{3} \frac{0!}{3!}$$

$$P(H_1 \cup H_2 \cup \cdots \cup H_n)$$

$\frac{(n-2)!}{n!}$

$$= P(H_1) + P(H_2) \cdots$$

$\frac{(n-1)!}{n!}$

$$- P(H_1 H_2) - \cdots$$

$$+ P(H_1 H_2 H_3)$$

$$- \cdots$$

$$= \binom{n}{1} \frac{(n-1)!}{n!} - \binom{n}{2} \frac{(n-2)!}{n!} + \binom{n}{3} \frac{(n-3)!}{n!} + \cdots$$

$$= \cancel{\frac{n!}{(n-1)!\,1!}}\,\cancel{\frac{(n-1)!}{n!}} - \cancel{\frac{n!}{(n-2)!\,2!}}\,\cancel{\frac{(n-2)!}{n!}} + \cdots$$

$$= \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \cdots \pm \frac{1}{n!}$$

$$= \sum_{i=1}^{n} \frac{(-1)^i}{i!} \underset{n\to\infty}{\approx} 1 - e^{-1} \approx 63\%$$

$$\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x \qquad e^{-1} = \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} \cdots$$

$$1 - e^{-1}$$

$$P(\text{no one}) = 1 - P(\text{someone}) = 1 - (1 - e^{-1}) = e^{-1}$$

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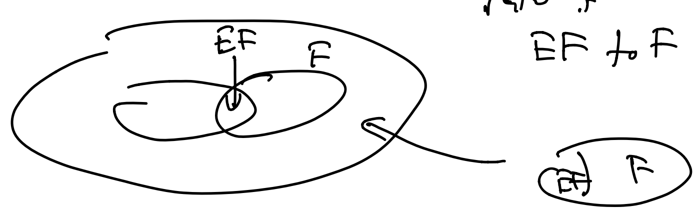
## Conditional Probability

$P(E|F)$ prob that E occurs, knowing that F occurs

$$= \frac{P(EF)}{P(F)}$$

rel. of EF to F



Side observation:

$$P(E^c|F) \overset{?}{=} 1-P(E|F)$$

to check: $P(E^c|F) = \dfrac{P(E^cF)}{P(F)} = \dfrac{P(F)-P(FE)}{P(F)}$

recall $P(F) = P(FE \cup FE^c)$ disjoint

$= P(FE)+P(FE^c)$

$$= \frac{P(F)}{P(F)} - \frac{P(FE)}{P(F)} = 1-P(E|F)$$

In practice, we may want to find $P(E)$ knowing only $P(E|F)$ or $P(E|F^c)$

ex: take a test to check if sick

if sick, test has 90% pos +
$S$ 10% neg

if healthy, test has 1% pos
99% neg

$P(+|S) = 9/10$ $P(+|S^c) = 1/100$

$P(+^c|S) = 1/10$ $P(+^c|S^c) = 99/100$

$P(+)$ ?

standard trick $\quad P(+) = P(+S) + P(+S^c)$

$$= P(+|S)P(S) + P(+|S^c)P(S^c)$$

So if we knew $P(S)$, would know $P(+)$

Typical problem might be $P(S|+)$

need to know $P(S)$ (without knowledge of test)

e.g. 1% chance of being sick.

$$P(S|+) = \frac{P(S+)}{P(+)} = \frac{P(S+)}{P(+|S)P(S) + P(+|S^c)P(S^c)}$$

$$= \frac{P(+|S)P(S)}{P(+|S)P(S) + P(+|S^c)P(S^c)}$$

$$= \frac{(9/10)(1/100)}{(9/10)(1/100) + (1/100)(99/100)}$$

$$= \frac{(9/10)}{(9/10) + (99/100)} \approx \frac{1}{2}$$

Repeat some useful moves:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

$$P(E) = P(EF) + P(EF^c)$$

$$\boxed{P(EF) = P(E|F)P(F)}$$

$$\boxed{P(E) = P(E|F)P(F) + P(E|F^c)P(F^c)}$$

$EF = FE \qquad P(EF) = P(FE)$

$$P(EF) = P(E|F)P(F)$$
$$= P(F|E)P(E)$$

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$H$ = hypothesis $\qquad E$ = evidence

$$P(H) \quad \text{vs.} \quad P(H|E) = \frac{P(HE)}{P(E)} = \frac{P(E|H)P(H)}{P(E)}$$

$$P(H|E) = P(H) \cdot \frac{P(E|H)}{P(E)}$$

$$= P(H) \cdot \frac{P(E|H)}{P(E|H)P(H) + P(E|H^c)P(H^c)}$$

## Odds (Convenient Shortcut)

**Def** odds of A $= P(A)/P(A^c)$

2-1 odds that A happens $\iff P(A) = 2P(A^c)$

$P(A) = \frac{2}{3}$

$P(A^c) = \frac{1}{3}$

$$\frac{P(H|E)}{P(H^c|E)} = \frac{P(H)\,P(E|H)/P(E)}{P(H^c)\,P(E|H^c)/P(E)} = \frac{P(H)}{P(H^c)}\,\frac{P(E|H)}{P(E|H^c)}$$

odds after evidence

odds get mult. by $\frac{P(E|H)}{P(E|H^c)}$

ex: S - patient is sick

T - test is positive

$P(S) = 6/10 \qquad P(S|T) = \,?$

$P(T|S) = 1 \qquad P(T|S^c) = 3/10$

$$\frac{P(S)}{P(S^c)} = \frac{(6/10)}{(4/10)} = \frac{3}{2} \qquad \frac{P(S|T)}{P(S^c|T)} = \frac{3}{2}\cdot\frac{\overbrace{P(T|S)}^{=1}}{\underbrace{P(T|S^c)}_{=3/10}}$$

$= 5$

$x = P(S|T) \qquad 1 - x = P(S^c|T)$

$\frac{x}{1-x} = 5 \qquad x = 5 - 5x \qquad 6x = 5$

$x = 5/6$

$= \frac{1}{2} + \frac{1}{3}$

$= .833\ldots$