Conditional Probability Recap

Suppose we have a population exposed to a disease

10% pop is resistant (10% chance of infection)
80% normal (60% chance of infection)
10% is susceptible (80% chance of infection)

suppose person A is infected
what is the prob that A is susceptible?

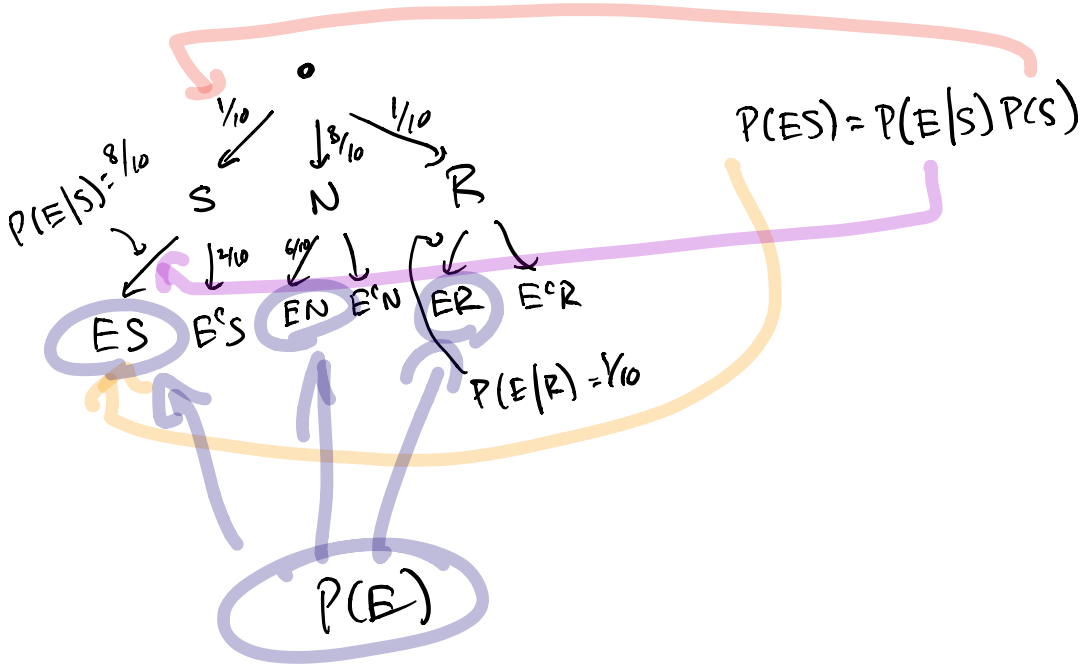
E infected R resistant N normal S susceptible

$$P(S|E) = \frac{P(SE)}{P(E)} = \frac{P(E|S)P(S)}{P(RE)+P(NE)+P(SE)}$$

$P(S|E)$ ↑ want

$E = (R \cup N \cup S)E = RE \cup NE \cup SE$

$$= \frac{P(E|S)P(S)}{P(E|R)P(R)+P(E|N)P(N)+P(E|S)P(S)}$$



Given a probability space S, P

and an event $E \subset S$, get a new probability function!

$$Q(F) = P(F|E)$$

Satisfies all same properties (of a probability fcn)

$F_1, F_2, \ldots$ disjoint, $Q(\cup F_i) = \sum Q(F_i)$

etc...

"Hw" $Q(G|F) \left(= P(G|F|E)\right)$ $P(G|EF)$

$$= \frac{P(EFG)}{P(E)P(F|E)} = \frac{P(EFG)}{P(EF)}"$$

Random Variables

Def If S, P is a prob. space, a random variable is a function $X : S \to \mathbb{R}$

Notation: $P(X = a) \equiv P(\{s \in S \mid X(s) = a\})$

$= P(X^{-1}(a))$

$P(X \leq a) = P(\{s \in S \mid X(s) \leq a\})$

etc.

---

examples flip a coin 3 times, X = # heads

$X(H,T,H) = 2$

$P(X = 2) = \frac{\#\{HHT, HTH, THH\}}{8} = 3/8$

$P(X \leq 1) = 4/8 = 1/2$

Cards labelled 1-10, choose 3, X = value of largest card

$P(X \leq 5)$? $= \frac{\binom{5}{3}}{\binom{10}{3}}$

Flip a coin until you get heads.

X = how many flips it took.

$$P(X=5) = \left(\frac{1}{2}\right)^5$$

Can think about random variable as defining a prob function with sample space $\mathbb{R}$.

$$P(X \in E) = P(\{a \in S \mid X(a) \in E\})$$

$E \subset \mathbb{R}$

Warning the rule $P(X \in E) = \sum_{a \in E} P(X=a)$ generally doesn't make sense.

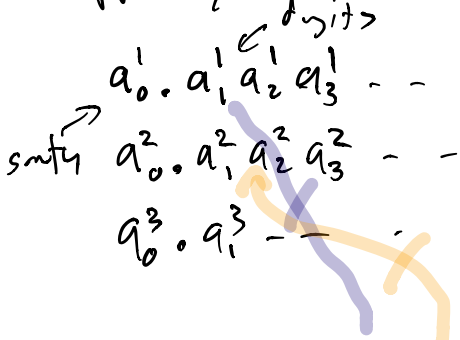
$\mathbb{R} = \bigcup_{a \in \mathbb{R}} \{a\}$ disjoint $P(E) = \sum_{a \in E} P(\{a\})$

the rule only applies to denumerable / countable collections of sets.

$E_1, E_2, E_3 \ldots$ disjoint $\Rightarrow$

$P(\cup E_i) = \sum P(E_i)$

Suppose you could list all real #s:



set $b_0 = 0$

$b_i \neq a_i^i, 0, 9$

$i > 0$

$b = 0 . b_1 b_2 b_3 \cdots$ is not on list.

can't list all real #s.

$S = \{1, 2, 3, \cdots\}$ $\quad P(\{i\}) = \left(\frac{1}{2}\right)^i$

fcn on $\mathbb{R}$'s:

$$\mathbb{R} = \bigcup_{\varepsilon} (\text{prob} > \varepsilon)$$