Math 477, Lecture 11 class work

Name: ____

Net ID: ____

1. How many ways can one distribute 3 identical red balls and 4 identical blue balls to 3 people?

   Solution. Since we are distributing both the red and blue balls, we can first count the number of ways to distribute the red and then multiply by the number of ways to distribute the blue balls.

   Using the trick of dividers, if we want to distribute n identical balls to 3 people, we can imagine a sequence of $n+2$ objects, either balls or dividers to represent the distribution of the balls to the 3 people. The number of ways to distribute the balls is then $\binom{n+2}{2}$. In particular, in our case there are $\binom{5}{2} = 10$ ways to distribute the red balls, and $\binom{6}{2} = 15$ ways to distribute the blue balls, for a total of $\binom{5}{2}\binom{6}{2} = 150$ ways to distibute all the balls. □

   If every distribution is equally likely, what is the probability that each person gets exactly 1 red ball?

   Solution. Since this question only depends on the distribution of the red balls, we need only ask what the probability that the red balls are distributed in this way. Since this represents exactly one of the $\binom{5}{2} = 10$ equally likely ways to distribute the red balls, the answer is 1/10. □

2. Suppose that a batch of 100 items contains 6 that have manufacturing errors and 94 that are made correctly.

   (a) If X is the number of tems with manufacturing errors in a randomly drawn sample of 10 items from the batch, find $P(X = 0)$ and $P(X < 2)$.

   Solution. Since there are $\binom{94}{10}$ ways to select 10 items which are each made correctly, and $\binom{100}{10}$ ways to select 10 items, $P(X = 0) = \frac{\binom{94}{10}}{\binom{100}{10}}$. The number of ways to select exactly one defective item is $\binom{6}{1}\binom{94}{9}$, and so

   $$P(X < 2) = P(X = 0) + P(X = 1) = \frac{\binom{94}{10}}{\binom{100}{10}} + \frac{\binom{6}{1}\binom{94}{9}}{\binom{100}{10}}.$$

   □

   (b) Suppose that the items with manufacturing errors have a 50% chance of functioning correctly, and other items have a 100% chance of functioning correctly. If a randomly chosen item functions correctly, what is the probability that it has manufacturing errors?

Sketch of Solution. Let E be the event that the selected item has a manufacturing error, and C the event that it functions correctly. We have $P(C|E) = 1/2$, $P(C|E^c) = 1$, $P(E) = 6/100 = 3/50$. We want to calculate $P(E|C) = P(EC)/P(C)$. We can calculate this as follows, using the things we know:

$$P(EC) = P(C|E)P(E),$$

and

$$P(C) = P(CE) + P(CE^c) = P(C|E)P(E) + P(C|E^c)E^c.$$

Now, just fill these quantities in, and we are done! □