Math 477, Lecture 3 class work

Name: ______________________________

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1. Suppose S is a sample space with subsets A, B, and such that $P(A+B) = 0.7$, $P(A) = P(B) = 0.5$. What is $P(AB^c)$? What is $P(AB)$?

   By the inclusion-exclusion principle, we have $P(A \cup B) = P(A) + P(B) - P(AB)$. Since $P(A \cup B) = 0.7, P(A) = 0.5 = P(B)$, we have $P(AB) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.5 - 0.7 = 0.3$. Since $P(A) = P(AB) + P(AB^c)$ (see notes from class), and since $P(A) = 0.5, P(AB) = 0.3$, we have $P(AB^c) = P(A) - P(AB) = 0.5 - 0.3 = 0.2$.

2. Suppose S is a sample space with subsets A, B, C, and such that $P(A) = 0.4, P(B) = 0.5, P(C) = 0.4, P(AB) = 0.2, P(AC) = 0.2, P(BC) = 0.1, P(ABC) = 0.1$, what is $P(A \cup B \cup C)$? What is $P((A \cup B)C^c)$?

   We have $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC) = 0.4 + 0.5 + 0.4 - 0.2 - 0.2 - 0.1 + 0.1 = 0.9$.

3. Suppose we roll a die 5 times. What is the probability that we get exactly 2 ones?

   Let S be the set of sequences of results of 5 rolls. Each sequence is equally likely. The number of sequences having exactly 2 ones is

$$\binom{5}{2}5^3$$

   Since there are $\binom{5}{2}$ possible choices for which 2 rolls result in ones, and 5^3 possible results for the other 3 rolls. Since there are 6^5 possible sequences of rolls all together, the probability is:

$$\frac{\binom{5}{2}5^3}{6^5}$$

4. Suppose that we are dealt 5 cards from a standard deck of 52 cards. What is the probability that we get a pair, but not three or four of a kind?

Quick reminder: each card in the deck has a value: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A, and a suit: H, D, C, S. There are 13 values and 4 suits for a total of $13 \times 4 = 52$ possible cards. Two cards are called a pair if they have the same value (but different suits). Three (or four) of a kind means that three (or four) cards have the same value (but different suits).

solution 1: Let S be the sample space consisting of all possible hands of 5 cards. There are $\binom{52}{5}$ possible hands, and each is equally likely. The number of hands which have exactly one pair is:

$$13\binom{4}{2}\binom{12}{3}4^3$$

Where $13\binom{4}{2}$ is the number of pairs (both cards share one of 13 possible values, and then we choose 2 of 4 possible suits), and $\binom{12}{3}4^3$ is the number of ways of choosing the remaining 3 cards (choose 3 of the 12 possible remaining values, and assign 4 possible suits to each of these 3 cards). This gives the probability as

$$\frac{13\binom{4}{2}\binom{12}{3}4^3}{\binom{52}{5}}$$

solution 2: Let S be the sample space consisting of sequences of 5 cards (dealt in a particular order). There are $(52)(51)(50)(49)(48) = \frac{52!}{47!}$ such hands, and each is equally likely. The number of hands which have exactly one pair is:

$$\binom{5}{2}(13)(4)(3)(12)(11)(10)4^3$$

Where $\binom{5}{2}$ represents the ways to choose which 2 of the 5 cards form the pair, 13 is the number of possibilities for the shared value of the pair, the 4 represents the number of possible suits for the first member of the pair and the 3 represents the number of values for the second member of the pair. Now, the remaining cards must have distince values and different values from the pair, so there are 12 possibilities for the value of the first non-pair card in the sequence, 11 possiblities for the second and 10 for the third. After choosing these, there are 4 possibilities for the suit of each of these, giving the 4^3 factor.

This gives the probability of:

$$\frac{\binom{5}{2}(13)(4)(3)(12)(11)(10)4^3}{\left(\frac{52!}{47!}\right)} = \frac{\frac{5!}{2!3!}(13)\frac{4!}{2!}\frac{12!}{9!}4^3}{\left(\frac{52!}{47!}\right)} = \frac{(13)\left(\frac{4!}{2!2!}\right)\left(\frac{12!}{9!3!}\right)4^3}{\left(\frac{52!}{47!5!}\right)} = \frac{13\binom{4}{2}\binom{12}{3}4^3}{\binom{52}{5}}$$

which is the same as we got before.