Math 477, Lecture 6 class work

1. If two regular dice are rolled, what's the probability that at least one lands on a 5, if we know that they both land on different numbers?

   Let E be the event that one lands on a 5, and F the event that they both land on different numbers. We want to calculate $P(F|E)$. By definition, we have $P(F|E) = P(EF)/P(E)$. Since E is a disjoint union:

   $$\{(5,i)|i \neq 5\} \cup \{(i,5)|i \neq 5\} \cup \{(5,5)\}$$

   we see $P(E) = \#E/\#S = (5+5+1)/36 = 11/36$ and since EF is the equal to the disjoint union

   $$\{(5,i)|i \neq 5\} \cup \{(i,5)|i \neq 5\}$$

   we see $P(EF) = \#EF/\#S = 10/36$. Therefore

   $$P(F|E) = P(EF)/P(E) = 10/11.$$

2. Two numbers a and b are chosen randomly from the set $\{1,2,3,4,5,6\}$ each result being equally likely. Let A be the event that $a+b=7$, and let B be the event that a and b have different parity (that is, one is even and the other is odd). Are A and B independent?

   Counting, we see $P(A) = 6/36 = 1/6$, $P(B) = 1/2$ and $AB = A$, since if two numbers have the same parity their sum is even and therefore can't be 7. Therefore $P(A)P(B) = 1/12 \neq P(AB) = P(A) = 1/6$. Therefore the events are not independent.

3. Two numbers a and b are chosen randomly from the set $\{1,2,3,4,5,6\}$ each result being equally likely. Let A be the event that a is odd, and let B be the event that a and b have different parity (that is, one is even and the other is odd). Are A and B independent?

   We can check that $P(A) = 1/2 = P(B) = P(AB)$. Therefore since $P(A)P(B) = 1/4 \neq 1/2 = P(AB)$, we see that the events are not independent.

4. If a person has a 9/10 chance of a getting a problem right on a given assignment, assuming that the performance on each problem is independent, and that there are 4 problems, what is the probability that the person gets exactly 3 problems correct?

   Let $E_{i,j,k}$ be the event that a person gets only problems i,j,k correct (here, $i,j,k \in \{1,2,3,4\}$). Then these give $4 = \binom{4}{3}$ mutually exclusive events whose union E is the event that a person gets exactly 3 problems correct. We have $P(E_{i,j,k}) = (1/10)(9/10)^3$, and so $P(E) = 4(1/10)(9/10)^3 = (2/5)(273/1000) = 546/5000 \sim 1/10$.

5. If a person has a 9/10 chance of a getting a problem right on a given assignment, assuming that the performance on each problem is independent, and that there are 4 problems, what is the probability that the person gets exactly 2 problems correct?

   Let $E_{i,j}$ be the event that a person gets only problems i,j correct (here, $i,j \in \{1,2,3,4\}$). Then these give $6 = \binom{4}{2}$ mutually exclusive events whose union E is the event that a person gets exactly 2 problems correct. We have $P(E_{i,j}) = (1/10)^2(9/10)^2$, and so $P(E) = 6(1/10)^2(9/10)^2 = (3/5)(1/10)(81/100) = 273/5000 \sim 1/20$.