

RESEARCH STATEMENT

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My research is in algebra and algebraic arithmetic geometry, particularly focusing on topics which relate to field arithmetic, noncommutative algebra and the Brauer group. My methods have primarily involved the use of tools and techniques from algebraic geometry, and ideas from analysis and topology in order to obtain results in algebra and field arithmetic.

My recent research has focused on the following topics:

1. *Finite dimensional algebraic structures*: central simple algebras, quadratic forms and algebraic with involution, and more generally, torsors for linear algebraic groups;
2. *Arithmetic of general fields*: field invariants, such as Galois cohomology groups, period-index problems, symbol length, generic splitting fields, Diophantine and cohomological dimensions;
3. *Arithmetic of semiglobal fields*: field patching, local-global principles for algebraic structures over such fields;
4. *Moduli stacks and derived categories of sheaves and twisted sheaves*: derived equivalences, index reduction for Brauer classes, Ulrich bundles, arithmetic of genus 1 curves, Weil-Châtelet groups of Abelian varieties;

and these areas represent areas of active inquiry of my current research as well. Before discussing these topics in a little bit more detail below, let me briefly mention what other things I am thinking about, in my work in progress. My other current projects involve topics in noncommutative geometry (specifically, noncommutative birational geometry), algebraic classifying spaces and higher stacks, splitting fields of division algebras and cohomology classes, parameter spaces for Galois extensions, computing Chow groups of homogeneous varieties, Fulton-MacPherson configuration spaces, norm varieties and algebraic cobordism, and new abstract contexts for local-global principles.

Finite dimensional algebraic structures were the original focus of my research. I began my studies by investigating the use of methods from algebraic geometry in the study of finite dimensional division algebras.

Division algebras, and more generally, central simple algebras, turn out to have critical importance in a number of areas: relating to the Tate conjecture and the Tate-Shafarevich groups, obstructions to local-global principles, problems of rationality in algebraic geometry, gerbes and hence obstruction classes for various naturally arising moduli stacks.

My research on central simple algebras and related structures has used, as its main method, the development and enrichment of dictionaries between these structures and the algebraic geometry of homogeneous varieties. In this way, more novel approaches in arithmetic and algebraic geometry may be brought to bear on the problems in algebra.

My contributions in this area include:

- new cases of and generalizations of a conjecture on Amitsur from the 1950's, concerning the birational equivalence of Severi-Brauer varieties [Kra03, Kra08];
- results on common splitting fields of collections of central simple algebras, and splitting fields for separable algebras, generalizing classical results of Albert and Risman [Kra10a];
- results on when one can distinguish division algebras by their finite splitting fields (joint with Kelly McKinnie) [KM15];
- determination of the Chow group of zero cycles on certain homogeneous varieties [Kra10c];
- descriptions of the Chow Motives of unitary and orthogonal homogeneous varieties [Kra07];
- work on birational geometry of symmetric powers of varieties, particularly Severi-Brauer varieties (joint with David J. Saltman) [KS04];
- description of torsion in Chow groups of certain generalized Severi-Brauer varieties (joint with Caroline Junkins and Nicole Lemire) [JKL17].

The arithmetic of general fields has seen vast advances in recent years, with the advent of motivic homotopy theory and the solutions of the Milnor and Bloch-Kato conjectures (see [MV99, Voe03a, Voe03b, Voe11, Wei09, OVV07]). These powerful theorems have led to more refined questions about Galois Cohomology (such as the symbol length problem), and have also lent us new methods by which we may attempt to answer them.

The problem of determining symbol length and indices for cohomology classes, and of constructing generic splitting varieties, is a topic of central importance in field arithmetic and ties in with other work in essential dimension of algebraic stacks (see for example, [BRV10, Kah00, KM03, CM13, CM14]).

My work in this area has included:

- clarifying the relationship between different measures of dimension of fields and complexity of cohomology classes, and in particular, showing that the index of a cohomology class may be bounded in terms of the Diophantine dimension [Kra16];
- giving explicit linear bounds on the p -cohomological dimension of a field in terms of its Diophantine dimension, answering a question of Serre in certain cases (joint with Eli Matzri) [KM15].

A semiglobal field is the function field of a curve over a complete discretely valued ground field. **The arithmetic of semiglobal fields** has been studied in a number of contexts; in order to understand higher class field theory [Kat86], in its relationship to Tate-Shafarevich groups [Gro68, PS96], and as an important ingredient in understanding torsors over finitely generated fields via local-global principles [PP03, LPS14]. As semiglobal fields are naturally the function fields of two-dimensional schemes, algebraic structures over semiglobal fields have been of great interest as an important step to understanding the behavior of such objects in more general higher-dimensional contexts.

Field patching is a method for studying the arithmetic of semiglobal fields, introduced initially by Harbater and Hartmann in 2006, and developed subsequently by myself, Harbater and Hartmann jointly from 2007. These methods have been featured in a number of past and upcoming workshops and conferences. For example, a summer school program in Heidelberg in 2008, an AIM workshop in 2011, the 2012 Arizona Winter School, a 2012 workshop in Essen, Germany, a workshop in 2015 at the University of Pennsylvania (“Local-global principles and their obstructions”), and an upcoming workshop in 2016 at Emory University (“Emory conference on higher obstructions and rational points”).

My work on semiglobal fields and field patching includes:

- obtaining bounds for u-invariants, and solutions to the period index problem for various semiglobal fields (joint with David Harbater and Julia Hartmann) [HHK09];
- the admissibility problem for semiglobal fields – which groups can be realized as Galois groups of maximal subfields of division algebras, (originally posed by M. Schacher for global fields in [Sch68], see also [FS95, Fei02, NP10, RS13]), (joint with David Harbater and Julia Hartmann) [HHK11];
- development of local-global principles for torsors for linear algebraic groups and Galois cohomology classes over semiglobal fields (joint with David Harbater and Julia Hartmann) [HHK15, HHK14];
- extending local-global principles from rational to retract rational linear algebraic groups over semiglobal fields [Kra10b];
- obtaining descriptions of Galois groups of semiglobal fields (joint with David Harbater, Julia Hartmann, R. Parimala and V. Suresh) [HHK⁺19];
- development of local-global principles for zerocycles of degree 1 on certain varieties over semiglobal fields (joint with Jean-Louis Colliot-Thélène, David Harbater, Julia Hartmann, R. Parimala and V. Suresh) [CTHH⁺19]
- computing obstructions for local global principles for algebraic tori over semiglobal fields [CTHH⁺20]
- considering local-global principles for relative curves over semiglobal fields (joint with David Harbater and Alena Pirutka) [HKP21]

Moduli stacks and derived categories of sheaves and twisted sheaves have enjoyed a rich connection to the theory of the Brauer group. The somewhat modern perspective that Brauer classes arise naturally as obstruction classes for the fine-ness of various moduli spaces of sheaves can be seen implicitly in many places, from Tate’s pairing [Tat58], to Grothendieck’s study of genus 1 fibrations [Gro68], and in explicit form in more modern references such as [HS05, Kha05, BBGN07, Lie13]. These natural obstruction classes have, in certain cases, also been shown to be realizable by generalized Clifford algebras [Hai84, Hai92, Kul03, CKM12], providing particularly concrete descriptions in these situations.

My work in this area includes:

- giving bounds for the index of a Brauer class over the function field of a p -adic surface (joint with Ben Antieau, Asher Auel, Colin Ingalls and Max Lieblich) [AAI⁺19]

- giving a generalized notion of a Clifford algebra associated to finite morphism, with applications to the period-index problem for genus 1 curves (joint with Max Lieblich) (in revision in the Journal of algebra; preprint: <https://arxiv.org/abs/1509.07195>);
- determining when the derived categories for torsors for Abelian varieties are equivalent, via DG-categorical methods (joint with Benjamin Antieau and Matthew Ward), [AKW17];
- providing new interpretations of the Tate pairing, and algorithms for computing Relative Brauer groups of genus 1 curves, (joint with Mirela Ciperiani) [CK12];
- computing index reduction formulas for Brauer classes with respect to function fields of genus 1 curves, using moduli spaces of twisted sheaves and their obstruction classes (joint with Max Lieblich, with an appendix by Bhargav Bhatt) [KL08].

REFERENCES

- [AAI⁺19] Benjamin Antieau, Asher Auel, Colin Ingalls, Daniel Krashen, and Max Lieblich. Period-index bounds for arithmetic threefolds. *Invent. Math.*, 216(2):301–335, 2019.
- [AKW17] Benjamin Antieau, Daniel Krashen, and Matthew Ward. Derived categories of torsors for abelian schemes. *Adv. Math.*, 306:1–23, 2017.
- [BBGN07] Vikraman Balaji, Indranil Biswas, Ofer Gabber, and Donihakkalu S. Nagaraj. Brauer obstruction for a universal vector bundle. *C. R. Math. Acad. Sci. Paris*, 345(5):265–268, 2007.
- [BRV10] Patrick Brosnan, Zinovy Reichstein, and Angelo Vistoli. Essential dimension, spinor groups, and quadratic forms. *Ann. of Math. (2)*, 171(1):533–544, 2010.
- [CK12] Mirela Ciperiani and Daniel Krashen. Relative Brauer groups of genus 1 curves. *Israel J. Math.*, 192(2):921–949, 2012.
- [CKM12] Emre Coskun, Rajesh S. Kulkarni, and Yusuf Mustopa. Pfaffian quartic surfaces and representations of Clifford algebras. *Doc. Math.*, 17:1003–1028, 2012.
- [CM13] Vladimir Chernousov and Alexander Merkurjev. Essential p -dimension of split simple groups of type A_n . *Math. Ann.*, 357(1):1–10, 2013.
- [CM14] Vladimir Chernousov and Alexander Merkurjev. Essential dimension of spinor and Clifford groups. *Algebra Number Theory*, 8(2):457–472, 2014.
- [CTHH⁺19] J.-L. Colliot-Thélène, D. Harbater, J. Hartmann, D. Krashen, R. Parimala, and V. Suresh. Local-global principles for zero-cycles on homogeneous spaces over arithmetic function fields. *Trans. Amer. Math. Soc.*, 372(8):5263–5286, 2019.
- [CTHH⁺20] J.-L. Colliot-Thélène, D. Harbater, J. Hartmann, D. Krashen, R. Parimala, and V. Suresh. Local-global principles for tori over arithmetic curves. *Algebraic Geometry*, 7(5):607–633, 2020.
- [Fei02] Walter Feit. $SL(2, 11)$ is \mathbb{Q} -admissible. *J. Algebra*, 257(2):244–248, 2002.
- [FS95] Burton Fein and Murray Schacher. $\mathbb{Q}(t)$ and $\mathbb{Q}((t))$ -admissibility of groups of odd order. *Proc. Amer. Math. Soc.*, 123(6):1639–1645, 1995.
- [Gro68] Alexander Grothendieck. Le groupe de Brauer. III. Exemples et compléments. In *Dix exposés sur la cohomologie des schémas*, volume 3 of *Adv. Stud. Pure Math.*, pages 88–188. North-Holland, Amsterdam, 1968.
- [Hai84] Darrell E. Haile. On the Clifford algebra of a binary cubic form. *Amer. J. Math.*, 106(6):1269–1280, 1984.
- [Hai92] Darrell E. Haile. When is the Clifford algebra of a binary cubic form split? *J. Algebra*, 146(2):514–520, 1992.
- [HHK09] David Harbater, Julia Hartmann, and Daniel Krashen. Applications of patching to quadratic forms and central simple algebras. *Invent. Math.*, 178(2):231–263, 2009.
- [HHK11] David Harbater, Julia Hartmann, and Daniel Krashen. Patching subfields of division algebras. *Trans. Amer. Math. Soc.*, 363(6):3335–3349, 2011.

- [HHK14] David Harbater, Julia Hartmann, and Daniel Krashen. Local-global principles for Galois cohomology. *Comment. Math. Helv.*, 89(1):215–253, 2014.
- [HHK15] David Harbater, Julia Hartmann, and Daniel Krashen. Local-global principles for torsors over arithmetic curves. *Amer. J. Math.*, 137(6):1559–1612, 2015.
- [HHK⁺19] David Harbater, Julia Hartmann, Daniel Krashen, Raman Parimala, and Venapally Suresh. Local-global Galois theory of arithmetic function fields. *Israel J. Math.*, 232(2):849–882, 2019.
- [HKP21] D. Harbater, D. Krashen, and A. Pirutka. Local–global principles for curves over semi-global fields. *Bull. Lond. Math. Soc.*, 53(1):177–193, 2021.
- [HS05] Daniel Huybrechts and Paolo Stellari. Equivalences of twisted $K3$ surfaces. *Math. Ann.*, 332(4):901–936, 2005.
- [JKL17] C. Junkins, D. Krashen, and N. Lemire. Schubert cycles and subvarieties of generalized Severi-Brauer varieties. *ArXiv e-prints*, April 2017.
- [Kah00] Bruno Kahn. Comparison of some field invariants. *J. Algebra*, 232(2):485–492, 2000.
- [Kat86] Kazuya Kato. A Hasse principle for two-dimensional global fields. *J. Reine Angew. Math.*, 366:142–183, 1986. With an appendix by Jean-Louis Colliot-Thélène.
- [Kha05] Madeeha Khalid. On $K3$ correspondences. *J. Reine Angew. Math.*, 589:57–78, 2005.
- [KL08] Daniel Krashen and Max Lieblich. Index reduction for Brauer classes via stable sheaves. *Int. Math. Res. Not. IMRN*, (8):Art. ID rnn010, 31, 2008.
- [KM03] Nikita Karpenko and Alexander Merkurjev. Essential dimension of quadrics. *Invent. Math.*, 153(2):361–372, 2003.
- [KM15] Daniel Krashen and Eliyahu Matzri. Diophantine and cohomological dimensions. *Proc. Amer. Math. Soc.*, 143(7):2779–2788, 2015.
- [Kra03] Daniel Krashen. Severi-Brauer varieties of semidirect product algebras. *Doc. Math.*, 8:527–546 (electronic), 2003.
- [Kra07] Daniel Krashen. Motives of unitary and orthogonal homogeneous varieties. *J. Algebra*, 318(1):135–139, 2007.
- [Kra08] Daniel Krashen. Birational maps between generalized Severi-Brauer varieties. *J. Pure Appl. Algebra*, 212(4):689–703, 2008.
- [Kra10a] Daniel Krashen. Corestrictions of algebras and splitting fields. *Trans. Amer. Math. Soc.*, 362(9):4781–4792, 2010.
- [Kra10b] Daniel Krashen. Field patching, factorization, and local-global principles. In *Quadratic forms, linear algebraic groups, and cohomology*, volume 18 of *Dev. Math.*, pages 57–82. Springer, New York, 2010.
- [Kra10c] Daniel Krashen. Zero cycles on homogeneous varieties. *Adv. Math.*, 223(6):2022–2048, 2010.
- [Kra16] Daniel Krashen. Period and index, symbol lengths, and generic splittings in Galois cohomology. *Bull. Lond. Math. Soc.*, 48(6):985–1000, 2016.
- [KS04] Daniel Krashen and David J. Saltman. Severi-Brauer varieties and symmetric powers. In *Algebraic transformation groups and algebraic varieties*, volume 132 of *Encyclopaedia Math. Sci.*, pages 59–70. Springer, Berlin, 2004.
- [Kul03] Rajesh S. Kulkarni. On the Clifford algebra of a binary form. *Trans. Amer. Math. Soc.*, 355(8):3181–3208 (electronic), 2003.
- [Lie13] Max Lieblich. On the ubiquity of twisted sheaves. In *Birational geometry, rational curves, and arithmetic*, pages 205–227. Springer, New York, 2013.
- [LPS14] Max Lieblich, R. Parimala, and V. Suresh. Colliot-Thelene’s conjecture and finiteness of u -invariants. *Math. Ann.*, 360(1-2):1–22, 2014.
- [MV99] Fabien Morel and Vladimir Voevodsky. \mathbf{A}^1 -homotopy theory of schemes. *Inst. Hautes Études Sci. Publ. Math.*, (90):45–143 (2001), 1999.
- [NP10] Danny Neftin and Elad Paran. Patching and admissibility over two-dimensional complete local domains. *Algebra Number Theory*, 4(6):743–762, 2010.
- [OVV07] D. Orlov, A. Vishik, and V. Voevodsky. An exact sequence for $K_*^M/2$ with applications to quadratic forms. *Ann. of Math. (2)*, 165(1):1–13, 2007.
- [PP03] R. Parimala and R. Preeti. Hasse principle for classical groups over function fields of curves over number fields. *J. Number Theory*, 101(1):151–184, 2003.

- [PS96] R. Parimala and R. Sujatha. Hasse principle for Witt groups of function fields with special reference to elliptic curves. *Duke Math. J.*, 85(3):555–582, 1996. With an appendix by J.-L. Colliot-Thélène.
- [RS13] B. Surendranath Reddy and V. Suresh. Admissibility of groups over function fields of p -adic curves. *Adv. Math.*, 237:316–330, 2013.
- [Sch68] Murray M. Schacher. Subfields of division rings. I. *J. Algebra*, 9:451–477, 1968.
- [Tat58] J. Tate. *WC-groups over p -adic fields*, volume 13 of *Séminaire Bourbaki; 10e année: 1957/1958. Textes des conférences; Exposés 152 à 168; 2e éd. corrigée, Exposé 156*. Secrétariat mathématique, Paris, 1958.
- [Voe03a] Vladimir Voevodsky. Motivic cohomology with $\mathbf{Z}/2$ -coefficients. *Publ. Math. Inst. Hautes Études Sci.*, (98):59–104, 2003.
- [Voe03b] Vladimir Voevodsky. Reduced power operations in motivic cohomology. *Publ. Math. Inst. Hautes Études Sci.*, (98):1–57, 2003.
- [Voe11] Vladimir Voevodsky. On motivic cohomology with \mathbf{Z}/l -coefficients. *Ann. of Math. (2)*, 174(1):401–438, 2011.
- [Wei09] C. Weibel. The norm residue isomorphism theorem. *J. Topol.*, 2(2):346–372, 2009.