

$$\mathbb{P}^n = \{ [a_0; \dots; a_n] \mid a_i \in k, \text{ not all } 0 \} = \{ \ell < k^{n+1} \mid \ell \text{ is a line} \}$$

$$[a_0; \dots; a_n] = [b_0; \dots; b_n]$$

if $\exists \lambda \in k^* \text{ s.t. } a_i = \lambda b_i$
all i

$$\mathcal{T} = \{ (v, \ell) \mid v \in k^n, \ell < k^n, \ell \perp v \}$$

$$= \{ (v_0, \dots, v_n), [a_0; \dots; a_n] \mid a_i = \lambda v_i \text{ same } \lambda \text{ all } i \}$$

$$\mathcal{T} \xrightarrow{\pi} \mathbb{P}^n$$

$$(v, \ell) \rightarrow \ell$$

sheaf \mathcal{M} on \mathbb{P}^n via

$$\mathcal{M}(U) = \{ f: U \rightarrow \mathcal{T} \mid \pi f = \text{id}_U \}$$

$\mathcal{M}(U)$ a module over $\mathcal{O}_{\mathbb{P}^n}(U)$

$$\left(\begin{array}{c} f_0(a_0; \dots; a_n), f_1(a_0; \dots; a_n) \dots \\ \text{s.t. } (f_0(\vec{a}), f_1(\vec{a}) \dots, f_n(\vec{a})) \\ [a_0; \dots; a_n] \\ \text{in } \mathcal{T} \end{array} \right)$$

$$\mathcal{T} = \{ (v_0, \dots, v_n) [a_0; \dots; a_n] \mid \dots \}$$

$$= \{ (\lambda a_0, \lambda a_1, \dots, \lambda a_n) [a_0; \dots; a_n] \}$$

sections defined by λ 's functions of a_i 's.

want $f_0(\vec{a}) h(\vec{a}) \dots$

and section given by

$$[a_0, \dots, a_n] \mapsto (f_0(\vec{a}) a_0, f_1(\vec{a}) a_1, \dots)$$

$$[a_0, \dots, a_n]$$

for this to make sense

$$e \rightarrow (h, p)$$

$$\frac{a_1 + a_2}{a_3}$$

$$\frac{f_i(\lambda \vec{a}) (\lambda a_i)}{\text{not the same as } f_i(\vec{a}) a_i}$$

in a.s.

sections of total space are subject to $\text{res. of } dy^{-1}$.

i.e. on $A_i^n \subset P^n$

$$\left\{ [a_0, \dots, a_{i-1}, 1, a_{i+1}, \dots, a_n] \right\}$$

coords $\frac{x_j}{x_i}$

then $\frac{1}{x_i}$ is a section of total space on this open set.

$$f_0 \hat{a}_0, f_1 \hat{a}_1, \dots$$

$$f_0(\lambda \vec{a}) \lambda a_0 \rightarrow \lambda a_0 f_0(\vec{a}) \lambda^{-1} \hat{a}_0$$

by 1

$$x_i$$

Recall: defined coherent, q -coh sheaves & \mathcal{O}_X -modules
 (only in Noeth case)

if $X = \text{Spec } A$, have an equiv of cats

$$\begin{array}{ccc} \{q\text{-coh sheaves on } X\} & \longleftrightarrow & \{A\text{-mods}\} \\ \tilde{M} & \longleftarrow & M \\ \mathcal{M} & \longrightarrow & \Gamma(X, \mathcal{M}) \end{array}$$

if A Noeth (coh on X) \longleftrightarrow {f.g. A -mods}

Prop 5.6: if X is affine then $\Gamma: \{q\text{-coh}/X\} \rightarrow \{A\text{-mods}\}$
 is exact.

Prop 5.7: For a general scheme.
 kernels, cokernels, images of morphisms of q -coh
 sheaves are q -coh. Also extensions of q -coh are
 q -coh. (Also all same for coherent)

At. h. extension part

if $\mathcal{F}', \mathcal{F}''$ are q -coh \mathcal{O}_X -mods and

$0 \rightarrow \mathcal{F}' \rightarrow \mathcal{F} \rightarrow \mathcal{F}'' \rightarrow 0$ is exact.

WTS \mathcal{F} is q -coh.

$\exists q$ -coh if $\mathcal{F}|_{\text{Spec } k}$
 is q -coh.

so wlog, restrict to case $X = \text{Spec } A$

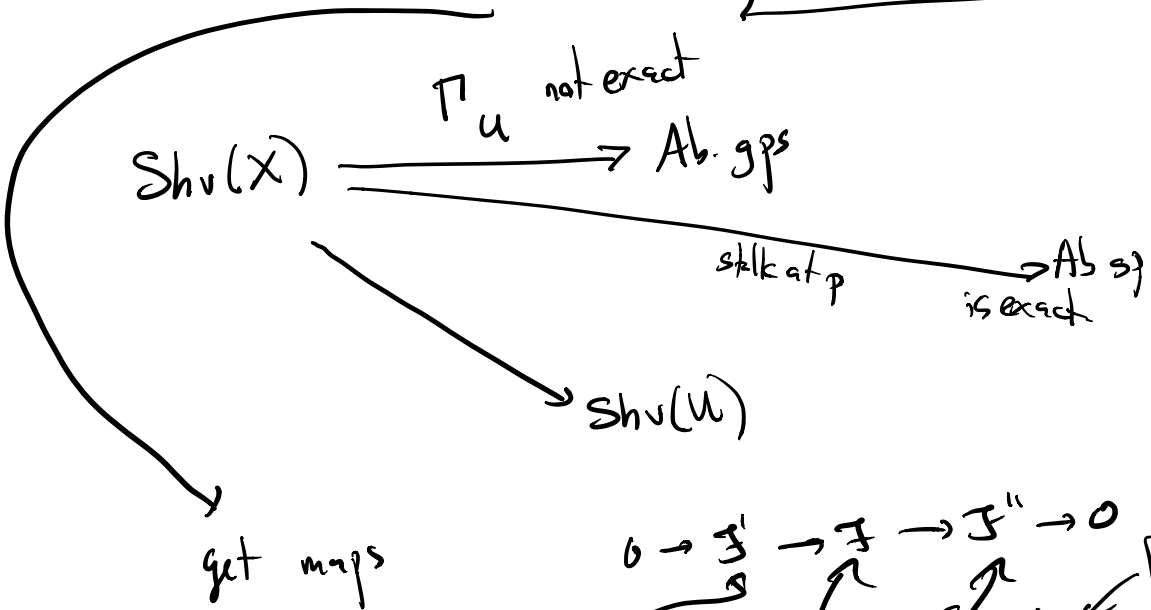
on $\text{Spec } A = X$,
 set $M = \Gamma(X, \mathcal{F})$

$$\begin{aligned} M' &\dots \mathcal{F}' \\ M'' &\dots \mathcal{F}'' \end{aligned}$$

by Ex S.3 \mathcal{F} nat. maps $\tilde{M}' \xrightarrow{\sim} \mathcal{F}'$
 $\tilde{M} \rightarrow \mathcal{F}$
 $\tilde{M}'' \xrightarrow{\sim} \mathcal{F}''$

subtext: (prop S.4)

- \mathcal{F} is g.coh \Leftrightarrow g.coh on an open cover
- \mathcal{F} g.coh on an affine $\Leftrightarrow \mathcal{F} \cong \tilde{M}$



$$\begin{array}{ccccccc} 0 & \rightarrow & \mathcal{F}' & \rightarrow & \mathcal{F} & \rightarrow & \mathcal{F}'' \rightarrow 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & M' & \rightarrow & M & \rightarrow & M'' \rightarrow 0 \end{array}$$

S-6

$$0 \rightarrow \tilde{M}' \rightarrow \tilde{M} \rightarrow \tilde{M}'' \rightarrow 0$$

$$\begin{array}{ccccccc} 0 & \rightarrow & \tilde{M}' & \rightarrow & \tilde{M} & \rightarrow & \tilde{M}'' \rightarrow 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & \mathcal{F}' & \rightarrow & \mathcal{F} & \rightarrow & \mathcal{F}'' \rightarrow 0 \end{array}$$

Actual 5.6: if $X = \text{Spec } A$, $0 \rightarrow \mathcal{F}' \rightarrow \mathcal{F} \rightarrow \mathcal{F}'' \rightarrow 0$
exact seq of \mathcal{O}_X -modules and \mathcal{F}' q -coh then

$\Gamma(\mathcal{F}) \rightarrow \Gamma(\mathcal{F}'')$ exact (and hence

$$0 \rightarrow \Gamma(\mathcal{F}') \rightarrow \Gamma(\mathcal{F}) \rightarrow \Gamma(\mathcal{F}'') \rightarrow 0$$

exact.)

Prop: 5.8 if $f: X \rightarrow Y$ a morphism then

• f^* of q -coh is q -coh

• if X, Y noeth, f^* of coh is coh

• if X noeth or f q -comp & sep then $f_* q$ -coh is q -coh.

when is f_* coh coherent? (correct answer: f proper)

Plan:

• Ideal sheaves

• Trusty sheaves

• coherence of f_* under projective morphisms

Def let Y be a closed subscheme of X ,
 $i: Y \rightarrow X$ inclusion define $\mathcal{O}_Y = \text{ker } i^\#$

Prop ^{5.9} \exists a bijection between \mathcal{O}_X -
 sheaves of ideals of \mathcal{O}_X and closed subschemes of X .



consider $\mathcal{O}_X/\mathcal{I}$

let $Y = \text{supp}(\mathcal{O}_X/\mathcal{I})$
 for this $= \{p \in X \mid (\mathcal{O}_X/\mathcal{I})_p \neq 0\}$

\mathcal{I} def. of $\mathcal{O}_X/\mathcal{I}$ $(A/\mathcal{I})_p \neq 0 \iff \mathcal{I} \not\subset p$

$A \longleftrightarrow \text{Spec } A$ pts \rightarrow primes $\cong \text{Spec } A$
 basic opens $D_f \cong \text{Spec } A_f$ $(A_f) = A_f$

$S \longleftrightarrow \text{Proj } S$ pts \rightarrow hom. primes (not containing irrelevant ideal $S_{>0}$)
 basic open $D_+^+(f) \cong \text{Spec } A(f)$

M an A mod

\tilde{M} sheaf of $\mathcal{O}_{\text{Spec } A}$ mods

$$\tilde{M}(D_f) = M \otimes_A A_f = M_f$$

N an S -mod
 \mathbb{Z} -graded

\tilde{N} sheaf of $\mathcal{O}_{\text{Proj } S}$ mods

$$\tilde{N}(D_+(f)) = N_{(f)}$$

= {hom elmts. of \mathcal{O} in

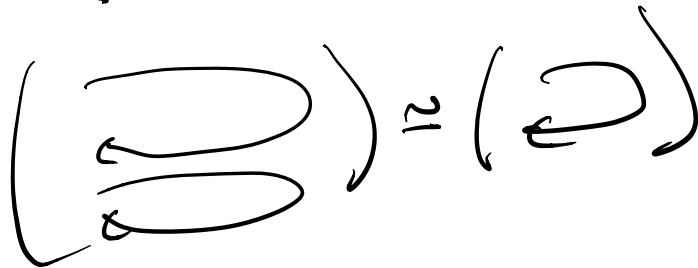
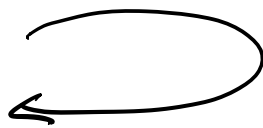
$N_{(f)}$ mod on $A_{(f)}$ }

N_f gen by hom elmts

f hom
$$d_f \frac{n}{f_i} = d_f n - \text{id}_f f$$



something.



$k_0 = \text{field of char } p \quad k = k_0(t)$

$$L = k(t^{1/p})$$

$$\begin{array}{c} \text{Spec } \mathbb{C} = X \\ | \\ \text{Spec } \mathbb{R} \end{array}$$

$$\begin{array}{c} \text{Spec } L = X \\ \downarrow \\ \text{Spec } k \end{array}$$

$$\begin{array}{c} X_{\bar{k}} = X \times_{\text{Spec } k} \text{Spec } \bar{k} \\ \text{Spec } k(t^{1/p}) \otimes_k \bar{k} \end{array}$$

$$X_{\mathbb{C}} \quad \mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \cong \frac{\mathbb{C}[x]}{x^2+1} \cong \frac{k(t^{1/p})}{x^p-t}$$

$$k(t^{1/p}) \otimes_k \bar{k} \cong \frac{\bar{k}[x]}{x^p-t}$$

choose $u \in \bar{k}$
 $u^p = t$

$$\frac{\mathbb{C}[x]}{(x+i)(x-i)} \cong \frac{\mathbb{C}[x]}{x+i} \oplus \frac{\mathbb{C}[x]}{x-i}$$

$$\frac{\bar{k}[x]}{x^p-u^p} = \frac{\bar{k}[x]}{(x-u)^p}$$

$$\cong \mathbb{C} \times \mathbb{C}$$

$$(0,1) \cdot (1,0) = 0$$

$$\begin{array}{l} x-u \neq 0 \\ (x-u)^p = 0 \end{array}$$