

# Algebraic Geometry 21/2 supplementary worksheet 5

## Proj, and properties of schemes

Critical Hartshorne problems in Chapter II

- (short term) 2.19, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6
  - (for later in semester) 3.7, 3.8, 3.10, 3.12
1. Let  $A$  be a commutative ring and consider the polynomial rings  $A[x]$ ,  $A[y]$ , and the ring  $B = A[x, y]/(xy - 1)$ . We may identify  $B$  with the localizations  $A[x]_x$  and  $A[y]_y$  via the natural inclusions. Let  $X$  be the scheme obtained by gluing  $\text{Spec } A[x]$  and  $\text{Spec } A[y]$  along  $\text{Spec } B$ , thought of as the open subschemes  $D_x$  and  $D_y$  in  $\text{Spec } A[x]$  and  $\text{Spec } B[x]$  respectively. Show that  $X \cong \text{Proj } A[s, t]$ .